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Understanding Labour Income Share Dynamics in Europe[*]

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Abstract

This paper seeks to understand labour share dynamics in Europe over the medium run. After documenting basic empirical regularities, we quantify the contribution of shifts in the sectoral and the employment composition of the economy to labour share movements. The findings from the shift-share analysis being on the descriptive side, we next identify the factors underlying labour share behaviour through a model-based approach. We proceed along the lines of Bentolila and Saint Paul (2003) but adopt a production function with capital-skill complementarity. We show that labour share movements are driven by a complex interplay of demand and supply conditions for capital and different skill categories of labour, the nature of technological progress and imperfect market structures. Based upon robust calibration, we show that most of the declining pattern in labour shares in nine EU15 Member States is governed by capital deepening in conjunction with capital-augmenting technical progress and labour substitution across skill categories. Although institutional factors also play a significant role, they appear to be of somewhat less importance. To illustrate the relevance of the technological explanation we quantitatively assess the dynamic impact of a permanent reduction in the fraction of unskilled employment on the labour share. We find that, for a given elasticity of substitution between skilled and unskilled labour, the more skilled labour is complementary to capital, the more pronounced the decline in the labour share.

Keywords: labour income share, medium term, two-level CES technology, market institutions.
JEL Classification: E25, J30, L51.

1. Introduction

The functional distribution of income shows how national income is divided among production factors. The distribution of increases in output between labour and capital has occupied the attention of the profession for decades. It has also been a subject of concern among policy makers and public opinion in recent years, where the observed decline in the labour share has been associated with important trends, such as globalisation, skilled-biased technological progress and changes in the institutional settings of labour and product markets.

The interplay between increases in output and factor income shares can be looked at from both the long- and the short-term perspective. The widespread belief among economists is that labour share movements over these two extreme time horizons can be neglected. On a secular basis, the relative stability of the labour share of income has acquired the condition of a "stylized fact". Gollin (2002), Gordon (2005), Piketty (2007), Piketty and Saez (2007), and Zuleta and Young (2007) all document trendless labour shares in the few countries for which long series are available, i.e. France, the UK and the US. In the context of the growth theory, the constancy of the labour share is associated with models that possess a steady state. Only when technology is Cobb-Douglas or else the production function is Constant-Elasticity-of- Substitution (CES) and all technical progress is labour-augmenting, does the neoclassical growth model deliver the convergence property with constant labour shares over the long run.

The conventional wisdom that oscillations in the labour share at business-cycle frequencies are irrelevant is more arguable. The increasing body of literature focussing on labour share movements in the short run proves that there is probably something to it. Notable pieces of work in the field of cyclical labour share movements for the US are represented by Young (2004), Hansen and Prescott (2005) and Ríos-Rull and Santaeulàlia-Llopis (2007).

In between the long- and the short-run there is the medium run, which is the focus of this paper. The medium run is probably the most relevant period from a policy perspective, yet the most difficult to deal with from a theoretical angle. Labour share movements over the medium run are often rationalised in terms of the transitional dynamics of a neoclassical growth model, which is governed by factor substitution, capital accumulation and the effect of technological progress, all of them operating at a time. Furthermore, the assumption of imperfect product and labour markets appears to be the more realistic in the medium term, which provides additional explanatory power to labour share movements. One should finally bear in mind that worldwide institutional changes, such as the globalisation process, tend to materialise over several decades.

This paper seeks to understand labour share movements in the former EU15 Mem-

ber States over the medium run. The paper is organised as follows. Section 2 documents empirical regularities and shows that over the past three decades labour shares have declined in many European countries. We then proceed to investigate this regularity along two different routes. Firstly, Section 3 analyses structural shifts in a descriptive manner, by quantifying the contribution of changes in the sectoral and the employment composition of the economy to labour share movements. The second approach, which we develop in Section 4, relies on a micro-founded model where the labour share is seen as a function of both technological and institutional parameters. The model is next used for calibration and simulation purposes in Section 5. Section 6 concludes with policy implications.

2. Medium-run empirical regularities

There is a vast literature that documents persistent movements in the labour share over the medium run. Two such studies focussing on a large number of countries include Harrison (2003) and Jones (2003). The evidence presented in this section for EU15 countries is consistent with this literature.

A very basic way to compute the labour income share simply entails dividing compensation of employees (CE_t) by gross value added at current basic prices (GVA_t):

$$LS_t^{ad} = \frac{CE_t}{GVA_t} \tag{1}$$

where LS_t^{ad} is the labour share calculated on the basis of national (i.e. aggregate) data on employees' remuneration and value added. The main drawback of expression (1) is that it ignores the labour income of proprietors. The self-employed typically earn a mix of capital and labour income, which are not identified separately in the National Accounts system. There is wide consensus that proprietors' labour should be remunerated at the average compensation of wage earners[1]. This assumption leaves us with the so-called "adjusted labour share":

$$ALS_t^{ad} = \frac{CE_t}{GVA_t} * \frac{TE_t}{E_t} \tag{2}$$

where TE_t and E_t respectively stand for total employment and employees. Scaling up the average compensation of wage earners for the entire workforce in the economy is a good approximation of self-employed labour income to the extent that they command the

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[1] See Gollin (2002).

same wage as employees. On the contrary, it is a poor assumption if there are systematic differences in labour income between employees and the self-employed. In particular, imputing the *national,* as opposed to the *sectoral,* average compensation to the self-employed distorts the measure of the labour share (Askenazy, 2003). Equation (2) does overestimate the income of the self-employed in the 1970s when they were mainly low income farmers (earning less than the average employee); similarly, it tends to underestimate their income in recent years as the majority of self-employed are high income earners (earning more than the average employee). A better estimate may thus be obtained by attributing to the self-employed the compensation of the average employee of their own activity branch. This way the national labour share is expressed as a weighted average across sectorally adjusted labour shares:

$$ALS_t^{sd} = \sum_{i=1}^{k} \frac{CE_{i,t} * TE_{i,t}}{va_{i,t} * E_{i,t}} = \sum_{i=1}^{k} \frac{va_{i,t}}{GVA_t} * \frac{CE_{i,t}}{va_{i,t}} * \frac{TE_{i,t}}{E_{i,t}} = \sum_{i=1}^{k} \omega_{i,t} * als_{i,t} \qquad (3)$$

where ALS_t^{sd} is the national labour share calculated on the basis of sectoral data, so that, for any sector i, $va_{i,t}, \omega_{i,t}$ and $als_{i,t}$ represent the gross value added at current basic prices, the sector's weight in national value added and the adjusted labour share. We argue in Appendix 1 that expression (3) is preferred to expressions (1) and (2) as a measure of the labour share. Table 1 and Table 2 report averages, the extreme values and the coefficient of variation of the labour share computed as in (3), respectively by country and by industry[2]. The data are taken from the EU KLEMS database[3]. The following facts emerge from these tables:

- In most countries the labour share reaches a maximum in the1970s to early 1980s and a minimum in the last years of the period. Only in Belgium and Portugal was this share lower in the 1970s than in the recent past.
- The volatility of the labour share is the highest in Ireland followed by a considerable distance by Finland, Sweden, Italy, France and Greece[4]. The labour share is the most

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[2]   Readers should be aware of the fact that descriptive statistics by industry reported in Table 2 exclude the observations of the labour share that exceed 1. This is the case of Agriculture, hunting, forestry and fishing in Austria and Portugal, Construction in Ireland and Hotels and restaurants in Belgium.This is due to the fact that the correction implied by (2) is not very reliable when the wages of the self-employed and the employees differ largely.

[3]   The EU KLEMS database includes measures of economic growth, productivity, employment creation, capital formation and technological change at the industry level for all European Union member states from 1970 onwards. The balance in academic, statistical and policy input in the EU KLEMS project is realised by the participation of 15 organisations from across the EU, representing a mix of academic institutions and national economic policy research institutes and with the support from various statistical offices and the OECD. The project is funded by the European Commission, Research Directorate.

[4]   The coefficient of variation in Austria is biased upwards because of a measurment error that arises when

Table 1 − Stylised Facts Of The Labour Share In The Medium Term, Country Perspective
Descriptive statistics by country, EU15 Member States, EU KLEMS data, 1970-2004

Country	pp. change 70-85	pp. change 86-95	pp. change 96-04	Mean	Maximum	(year)	Minimum	(year)	Coef. of varia- tion
BE	0.06	-0.01	-0.02	0.63	0.68	1980	0.58	1970	3.57
DE	-0.01	-0.02	-0.03	0.64	0.68	1981	0.58	2004	4.34
DK	-0.02	-0.04	0.01	0.62	0.66	1980	0.58	1994	3.54
EL	0.07	-0.09	-0.02	0.54	0.62	1982	0.48	2004	6.48
ES	-0.06	0.00	-0.01	0.60	0.64	1981	0.57	1989	4.16
FI	-0.07	-0.07	-0.04	0.62	0.73	1976	0.53	2002	9.73
FR	-0.04	-0.04	-0.01	0.63	0.69	1981	0.57	1998	6.77
IE	-0.17	-0.05	-0.07	0.59	0.77	1970	0.44	2002	14.18
IT	0.00	-0.05	-0.03	0.61	0.68	1975	0.53	2001	7.74
LU	-0.10	-0.01	0.02	0.55	0.62	1970	0.50	1999	6.32
NL	-0.08	0.01	0.00	0.61	0.68	1975	0.56	1985	5.57
AT	-0.13	-0.03	-0.06	0.68	0.80	1970	0.58	2004	9.20
PT	0.09	-0.03	0.00	0.62	0.69	1977	0.54	1970	5.62
SW	-0.09	-0.04	0.02	0.63	0.71	1977	0.55	1995	7.79
UK	-0.03	-0.03	0.03	0.67	0.73	1975	0.63	1996	3.43

Source: Own calculations of the basis of EU KLEMS data.

Note: Maximum/minimum: maximum/minimum value of the labour share in pp.

Coefficient of variation: standard deviation of labour share divided by mean, reported as a percentage.

stable in the United Kingdom, Denmark and Belgium.

- The largest pp. declines in the labour share were registered in Ireland, Austria, Luxembourg, Sweden and the Netherlands between 1970 and 1985 and Greece over 1986-1995.

- The labour share ranges on average from 0.39 in Electricity, gas and water supply, to 0.77 in Agriculture, hunting, forestry and fishing. The fact that the labour share varies more across industries than countries is suggestive of the importance of technological differences across industries.

the labour share is computed on the basis of (3). The imputation of the average wage in agriculture to the self-employed yields a value for the labour share in this industry above one for the overall sample. Given the relatively high weight of agriculture in the total economy in the early 1970s, the labour share calculated as in (3) is close to one at the beginning of the sample. This measurement error becomes less important at the end of the sample, because of the decreasing weight of Agriculture in the total value added.

Table 2 − Stylised Facts Of The Labour Share In The Medium Term, Industry Perspective

Descriptive statistics by industry, EU15 Member States, EU KLEMS data, 1970-2004

Industry	Mean	Maximum	(country)	Minimum	(country)	Coef. variation
Agriculture, hunting, forestry and fishing	0.77	0.97	DE	0.50	ES	45.52
Mining and quarrying	0.40	0.88	DE	0.07	NL	41.37
Total manufacturing	0.71	0.76	SW/UK	0.51	IE	9.99
Electricity, gas and water supply	0.39	0.56	IE	0.21	SW	22.69
Construction	0.74	0.92	DK	0.41	EL	18.69
Wholesale and retail trade	0.75	0.84	FR	0.55	EL	12.84
Hotels and restaurants	0.76	0.97	DE	0.46	EL	20.36
Transport, storage and communication	0.70	0.80	UK	0.55	FI	9.05
Finance, insurance, real estate and business services	0.41	0.59	UK	0.25	EL	21.68

Source: Own calculations on the basis of EU KLEMS data.

Note: Max./Min.: maximum/minimum value of the adjusted labour share in pp.

Coefficient of variation: standard deviation of labour share divided by mean, reported as a percentage.

3. A shift-share decomposition of medium-term movements in the labour share

The wage moderation of the last decade has been accompanied by a declining labour share, giving rise to distributional concerns. However, downward movements in the labour share may conceal important sectoral and employment developments. To illustrate the role of changes in the sectoral composition of the economy and in the composition of employment, this section pursues a shift-share decomposition of the labour share[5]. The analysis in this section is in much the same way as in De Serres *et al*. (2002), who also focus on the relevance of composition effects in accounting for the evolution of the labour share over time in OECD countries.

By first-differentiating expression (3), any change in the labour share is split into three components: i) the *sectoral composition effect*, which implies that a shift from high- to low-labour-share sectors will translate into an aggregate decline in the labour share, ceteris paribus; ii) the *employment structure effect*, according to which generalised reductions in the share of self-employed in total employment across sectors will result in a lower aggregate labour share, ceteris paribus[6]; and iii) the *employees' remuneration effect*, by which generalised reductions in the ratio of compensation of employees to value added across sectors lead to a lower aggregate labour share, ceteris paribus. In symbols,

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[5] For a detailed discussion on the shift-share decomposition of the labour share cfr. European Commission (2007).

[6] Intuitively, a lower share of self-employed in total employment implies, all other things being equal, that a lower level of compensation per employee is imputed to total employment.

$$\Delta ALS_t^{sd} = \sum_{i=1}^{k} \underbrace{\frac{CE_{i,t} * TE_{i,t}}{va_{i,t} * E_{i,t}} * \Delta\omega_{i,t}}_{Sectoral\ composition\ effect} + \underbrace{\omega_{i,0} * \frac{TE_{i,0}}{E_{i,0}} * \Delta\frac{CE_{i,t}}{va_{i,t}}}_{Employees'\ remuneration\ effect} - \underbrace{\frac{CE_{i,t}}{va_{i,t}} * \frac{1}{q_{i,t}} * \omega_{i,t} * \frac{\Delta q_{i,t}}{q_{i,0}}}_{Employment\ structure\ effect}$$

$$\text{with } q_{i,t} = \frac{E_{i,t}}{TE_{i,t}}. \tag{4}$$

Figure 1 displays the shift-share decomposition given by (4) for the former EU15 Member States. Notwithstanding the complexity and heterogeneity of labour share movements across countries, it is possible to identify some common patterns in the data:

- Over the period 1970-2004, the sectoral and the employment composition effects both contributed to a reduction in the aggregate labour share.
- The *employees' remuneration effect* has been sizeable during the sub-periods 1970-1985 and 1996-2004. Whether this effect contributed to a downward (due to wage moderation) rather than an upward (due to wage acceleration) movement in the labour share depends on the country.

The shift-share analysis reveals the importance of structural forces. To illustrate this more clearly, we construct a counterfactual labour share where the sectoral and the employment composition are set at their 1970 levels[7] (Figure 2). This allows the *employees' remuneration effect* to be disentangled from the other two structural sources of labour share movements. One can see that when the sectoral and the employment composition of the economy are kept constant, the labour share either remains broadly stable or declines at a slower pace.
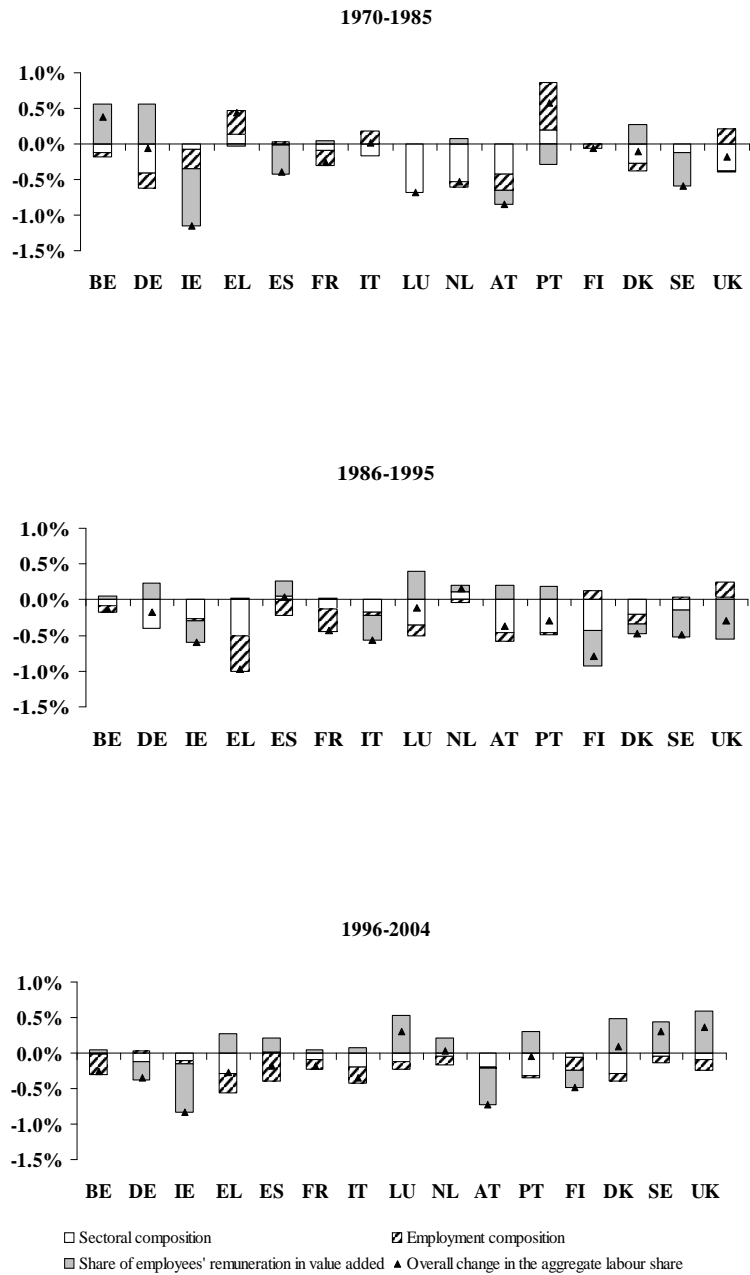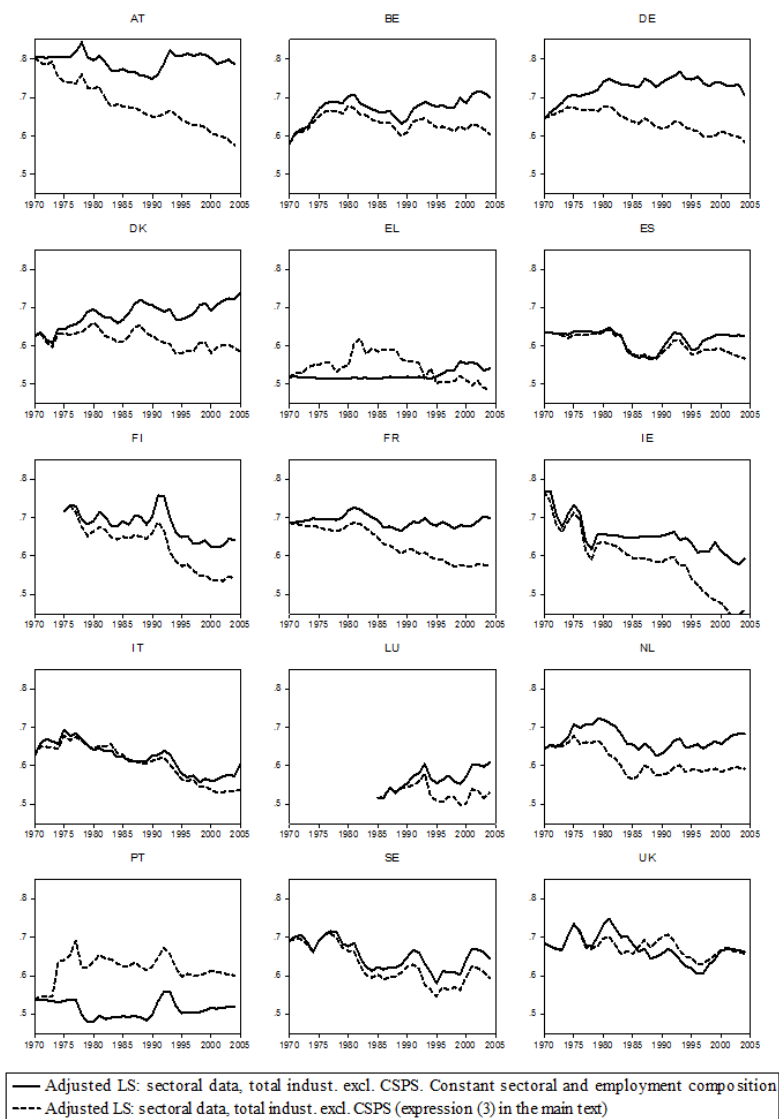
4.   Theoretical model

4.1   Static model

This section presents a model to account for labour share movements in the medium term. The modelling approach is sequential. We start by adopting a two-level CES technololgy with physical capital, skilled and unskilled labour. We subsequently incorporate intermediate inputs to the production function. The specification of the labour share is then modified to allow for imperfect competition in the goods market and bargaining in the labour market. Finally, we explore the role of labour hoarding to account for labour share movements. Expression (27) at the end of this section can be regarded as a general specification,

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[7]   For Finland and Luxembourg the sectoral and the employment composition are respectively set at their 1975 and 1985 levels.

Figure 1: *Sources of changes in the labour share in EU15 Member States.*
Average annual percentage change.

1970-1985

1986-1995

1996-2004

☐ Sectoral composition   ▨ Employment composition
▦ Share of employees' remuneration in value added   ▲ Overall change in the aggregate labour share

Figure 2: *Labour share (expression (3)) versus alternative labour share where the sectoral and employment composition are set at 1970 levels, EU15 Member States.*

Adjusted LS: sectoral data, total indust. excl. CSPS. Constant sectoral and employment composition
---- Adjusted LS: sectoral data, total indust. excl. CSPS (expression (3) in the main text)

from which nested versions may be obtained by imposing economically meaningful restrictions. Details on algebra are provided in Appendix 2.

A two-level CES technology with labour heterogeneity

Following Sato (1967), let us consider a "two-level CES production technology" with three inputs, namely, physical capital, skilled labour and unskilled labour. The "first level", given by a CES composite of physical capital and skilled labour, is nested with unskilled labour into another CES function, representing the "second level"[8]. Such an array of production possibilities is given by:

$$Y = \left[\alpha X^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(B_u L_u\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{5}$$

where

$$X = \left[a\left(AK\right)^{\frac{\eta-1}{\eta}} + (1-a)\left(B_s L_s\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{6}$$

In our notation, Y is output, X is the CES composite of physical capital and skilled labour and K, L_s, L_u stand for physical capital, skilled and unskilled labour. A production function such as (5) is characterized by three kinds of parameters: i) distribution parameters, α and $a \in (0,1)$; ii) efficiency parameters, A, B_u and B_s, representing technical progress specific to each input; and iii) substitution parameters, η and σ, respectively the elasticity of substitution between physical capital and skilled labour, and the elasticity of substitution between the composite input and unskilled labour[9]. It is assumed that $0 < \eta < \sigma < \infty$. $0 < \sigma < \infty$ implies that a reduction in the price of the composite X relative to unskilled labour will trigger some substitution against unskilled labour. The assumption $\eta < \sigma$ compares the ease of substitution of physical capital to the two skill categories. It implies higher complementarity (or less substitution) between physical capital and skilled labour than between the composite capital and unskilled labour. This is the well-known "capital-skill complementarity" hypothesis[10].

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[8] There is one point worth making in equation (5). Although there are two other possibilities of nesting the two-level CES function, the specification given by (5) is consistent with the empirical literature. Fallon and Layard (1975) and Krusell *et al.* (2000) present evidence in support of (5) and explain why including skilled labour and capital in the first level is the most plausible variant of nesting.

[9] The elasticity of substitution between capital and unskilled labour and between skilled labour and unskilled labour are both equal to σ.

[10] The hypothesis of "capital-skill complementarity" was first formalized by Griliches (1969). A related empirical literature has demonstrated that physical capital and skilled labour have been relatively complementary in

Under the assumption of perfect competition the labour share is given by:

$$LS_{PC} = \frac{w_u^{PC} L_u + w_s^{PC} L_s}{Y} = \frac{Y'_{L_u} L_u + Y'_{L_s} L_s}{Y} \tag{7}$$

where w_i and $Y'_{L_i}, i = u, s$, represent the real wage and the marginal productivity of each type of labour. Substituting in (7) the value added and marginal productivities consistent with a production function like (5), one can get the following expression for the labour share (see Appendix 2):

$$LS_{PC} = 1 - \frac{a}{(1-a)} (Ak)^{\rho} \left\{ \alpha^{\varepsilon} (1-a)^{\varepsilon-1} + (1-\alpha)^{\varepsilon} l^{\frac{\varepsilon-\sigma}{\sigma}} \omega^{\varepsilon-1} \right\}^{\frac{\sigma\rho}{\varepsilon(\sigma-1)}} \tag{8}$$

where LS_{PC} denotes the labour share under perfect competition, $\rho = \frac{\eta-1}{\eta}$ and $\varepsilon = \frac{\sigma\rho}{\sigma(\rho-1)+1}$. Equation (8) expresses the labour share as a function of $k = \frac{K}{Y}$, $l = \frac{B_s L_s}{B_u L_u}$ and $\omega = \frac{\frac{w_s^{PC}}{B_s}}{\frac{w_u^{PC}}{B_u}}$, which represent the capital-output ratio, the relative supply of skilled labour and the skill premium, all measured in efficiency units. Expressing the skill premium as a function of relative factor quantities, one may alternatively obtain the labour share in terms of input ratios:

$$LS_{PC} = 1 - a (Ak)^{\rho} \alpha^{\frac{\sigma\rho}{\sigma-1}} \left\{ 1 + \frac{1-\alpha}{\alpha} l^{\frac{1-\sigma}{\sigma}} \left[a \left(\frac{AK}{B_s L_s} \right)^{\rho} + (1-a) \right]^{\frac{1-\sigma}{\sigma\rho}} \right\}^{\frac{\sigma(\rho-1)+1}{\sigma-1}} \tag{9}$$

Unlike the case of homogeneous labour, where the labour share moves along a stable (non-linear) relationship with the capital-output ratio (see Bentolila and Saint Paul, 2003), labour heterogeneity introduces a shift factor, which depends on two input ratios: the relative supply of capital to skilled labour and the relative supply of skilled to unskilled labour.

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the past two centuries and are still so today. Goldin and Katz (1996) show that economy-wide capital and skilled labour complementarity emerged as a result of the adoption of several crucial technological advances, including the shift from the factory to continuous-process or batch methods, with electrification and the adoption of unit-drive machines reinforcing the change through the automation of hauling and conveying operations. Moreover, capital-skill complementarity is believed to be in full blossom today with ICT developments having a skill-biased component. Caselli and Coleman (2001) present robust findings that high levels of educational attainment are important determinants of computer-technology adoption. Krusell *et al.* (2000) show that capital-skill complementarity can be the source behind the increase in the US skilled premium. Briefly, empirical research indicates that new technologies tend to substitute for unskilled labour in the performance of routine tasks, while assisting skilled workers in executing qualified work.

The labour share specification given by (9) can be regarded as a general equation encompassing particular cases for specific values of the elasticities of substitution η and σ. Table 3 summarizes income shares accruing to the various inputs consistent with (5) for specific values of the elasticities of substitution η and σ. All income shares ($\theta_{Lu}, \theta_X, \theta_L, \theta_K, \theta_{Ls}$) are expressed as a function of relative factor quantities $\frac{AK}{B_u L_u}, \frac{AK}{B_s L_s}, l$.

The main observation to emerge from this table is that the relative supply of production factors affects income shares as long as the technology is different from Cobb-Douglas. Under case 1, both the inter- and intra-class elasticities of substitution are equal to one ($\sigma = \eta = 1$), thus all income shares are constant and given by the distribution parameters α and a. Under case 2, the elasticity between groups is equal to one ($\sigma = 1$) and the elasticity within groups is lower than one ($\eta < 1$), implying complementarity between capital and skilled labour. Consequently, the shares of the composite input and unskilled labour are determined by α. The remaining shares θ_L, θ_K and θ_{Lu} (which imply a separation of the factors within the composite) all depend on the ratios $\frac{B_s L_s}{AK}$ and $\frac{B_u L_u}{AK}$ through η. Under case 3, $\sigma > 1$, which explains why both $\frac{AK}{B_u L_u}$ and $\frac{B_s L_s}{B_u L_u}$ play a role in the division of income between the composite input and unskilled labour. Furthermore, because substitution of capital to skilled labour is as in Cobb-Douglas ($\eta = 1$), the labour share is only influenced by changes in the relative supply of unskilled labour $\frac{B_u L_u}{B_s L_s}$ (in a magnitude that depends of $\frac{1-\alpha}{\alpha}$ and σ). Finally, under the general case $\sigma > \eta$, one can see that the labour share is influenced by all three ratios $\frac{AK}{B_u L_u}, \frac{AK}{B_s L_s}, l$, the two distribution parameters α and a and the two elasticities of substitution σ and ρ.

To further illustrate the implications of changes in relative factor quantities, it is convenient to express the labour share in the following way:

$$LS_{PC} = \theta_{Lu} \left(1 + \frac{w_s^{PC}}{w_u^{PC}} \frac{L_s}{L_u} \right) \tag{10}$$

where $\theta_{Lu} = \frac{w_u^{PC} L_u}{Y}$ is the income share of unskilled labour. Note that expression (10) implies that the labour share is a function of relative wages, with the result that an increase in wage inequality raises the labour share. We show in Appendix 3 that $\theta_{Lu} = \theta_{Lu} \left(\frac{AK}{B_u L_u}, l \right)$ and that $\frac{w_s^{PC}}{w_u^{PC}} = \frac{w_s^{PC}}{w_u^{PC}} \left(\frac{AK}{B_u L_u}, \frac{AK}{B_s L_s}, l \right)$, meaning that the labour share can be expressed as a

sole function of input ratios[11]. Comparative static results are as follows (see Appendix 3).

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[11] Equation (9) expresses the labour share as a function of the relative supply of skilled labour, the capital-output ratio and the wage premium. Since the latter two variables can in turn be expressed as a function of relative factor quantities, it suffices to focus on these ratios to see the implications of the model for the labour share.

All other things being equal:

- The labour share responds positively to an increase in the capital to skilled labour ratio provided there is some substitution between capital and skilled labour, i.e. $\frac{\partial LS_{PC}}{\partial \left(\frac{AK}{B_s L_s} \right)} > 0$
  if and only if $\eta > 0$, which always holds, as η is strictly positive for all admissible values. With capital-skill complementarity, an increase in equipment per skilled worker increases the relative demand of qualified labour, thus leading to an increase in the wage premium (decreasing with η). In the new equilibrium, the income share of unskilled labour will remain unchanged while a larger number of skilled workers will be employed at a higher wage. Thus, the share of income accruing to labour will be higher.

- The labour share responds negatively to an increase in the relative supply of skilled workers if skilled and unskilled labour are highly substitutive, i.e. $\frac{\partial LS_{PC}}{\partial \left(\frac{L_s}{L_u} \right)} < 0$ if $\sigma > 1$.

  An increase in $\frac{L_s}{L_u}$ creates an excess of supply of qualified workers and a reduction in their relative wage. The higher σ, the more unskilled workers will be replaced by skilled labour in production and the sharper the fall in the unskilled's income share. Compared with the initial state of the economy, the relative demand for skilled workers will be higher, and both the labour share of the unskilled and the wage premium will be lower. If $\sigma > 1$, the overall effect on the labour share will be negative.

- Similarly, the labour share responds negatively to an increase in the capital to unskilled labour ratio if capital and unskilled labour are highly substitutive, i.e. $\frac{\partial LS_{PC}}{\partial \left(\frac{AK}{B_u L_u} \right)} < 0$ if and
  only if $\sigma > 1$. An increase in $\frac{AK}{B_u L_u}$ creates an excess of supply of equipment and a reduction in the price of capital in terms of unskilled labour. The substitution in production of capital for unskilled labour implies a reduction in the unskilled share in total income (increasing with σ). With capital-skill complementarity, the relative demand for skilled workers rises, which increases the wage premium. In the new equilibrium, both the relative demand for qualified labour and its relative price will be higher, while the labour income share of the unskilled will be lower. If $\sigma > 1$, the latter effect predominates and the labour share will decrease following a positive shock to capital.

- A sufficient condition for capital-augmenting technical progress to push the labour share downwards is that $1 < \eta < \sigma$, i.e. $\frac{\partial LS_{PC}}{\partial A} < 0$ if $1 < \eta < \sigma$.

Briefly, a technology with capital-skill complementarity like (5) can account for episodes where declining labour shares are accompanied by (some or all of) the following phenomena: i) capital-augmenting technical progress; ii) a reduction in the capital-skilled labour ratio, iii) unskilled labour becomes relatively scarce with respect to capital and/or skilled labour.

Table 3 − Factor Shares For Specific Values Of The Elasticities Of Substitution σ and η

	Case 1 $\sigma = 1; \eta = 1 \; (\rho = 0)$	Case 2 $\sigma = 1; \eta < 1 \; (\rho < 0)$
θ_{Lu}	$1 - \alpha$	$1 - \alpha$
θ_X	α	α
θ_L	$1 - \alpha a$	$1 - a \left(Ak \right)^\rho = 1 - \dfrac{a(AK)^\rho}{X^\alpha (B_u L_u)^{1-\alpha}} = 1 - \left(a \dfrac{1}{\left(a + (1-a)\left(\frac{B_s L_s}{AK} \right)^\rho \right)^\alpha \left(\frac{B_u L_u}{AK} \right)^{\rho(1-\alpha)}} \right)$
θ_K	αa	$a \left(Ak \right)^\rho = \dfrac{a(AK)^\rho}{X^\alpha (B_u L_u)^{1-\alpha}} = a \dfrac{1}{\left(a + (1-a)\left(\frac{B_s L_s}{AK} \right)^\rho \right)^\alpha \left(\frac{B_u L_u}{AK} \right)^{\rho(1-\alpha)}}$
θ_{Ls}	$(1 - a)\,\alpha$	$\alpha - a \left(Ak \right)^\rho = \alpha - \dfrac{a(AK)^\rho}{X^\alpha (B_u L_u)^{1-\alpha}} = \alpha - \left(a \dfrac{1}{\left(a + (1-a)\left(\frac{B_s L_s}{AK} \right)^\rho \right)^\alpha \left(\frac{B_u L_u}{AK} \right)^{\rho(1-\alpha)}} \right)$

Note: Production function is $Y = \left(AK \right)^{a\alpha} \left(B_s L_s \right)^{(1-a)\alpha} \left(B_u L_u \right)^{1-\alpha}$ in Case 1,

and $Y = \left(\left[a \left(AK \right)^\rho + (1-a)\left(B_s L_s \right)^\rho \right]^{\frac{1}{\rho}} \right)^\alpha \left(B_u L_u \right)^{1-\alpha}$ in Case 2.

A three-level CES technology with intermediate inputs

This section shows that the fraction of value added absorbed by labour is affected by changes in the relative price of intermediate inputs, which enter as an additional scaling factor in the relationship between the labour share and the capital output ratio. To see this formally, let us adopt the following CES production function for gross output ($\widetilde{Y}$):

$$\widetilde{Y} = \left\{ \gamma \left(\left[\alpha X^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) \left(B_u L_u \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right)^{\frac{\lambda-1}{\lambda}} + (1 - \gamma) I^{\frac{\lambda-1}{\lambda}} \right\}^{\frac{\lambda}{\lambda-1}} \tag{11}$$

where the capital-labour composite is now nested with intermediate inputs into a "third level" CES aggregator. I stands for intermediate inputs, $0 < \gamma < 1$ is a distribution parameter and $0 < \lambda < \infty$ determines the elasticity of substitution between intermediate inputs and the aggregator of primary inputs. Real value added is defined as (see Bruno and Sachs 1985, ch. 2, for a discussion):

$$Y = \widetilde{Y} - \frac{p_I}{\widetilde{p}} I \tag{12}$$

where $\widetilde{p}$ and p_I respectively denote the price deflators of gross output and the intermediate inputs, so $\frac{p_I}{\widetilde{p}}$ represents the real price of intermediate inputs in terms of gross output. We show in Appendix 2 that optimising behaviour by firms implies the following expression of the labour share:

Table 3 (cont.) − Factor Shares For Specific Values Of The Elasticities Of Substitution σ and η

	Case 3 $\sigma > 1; \eta = 1 (\rho = 0)$	Case 4 (general case) $\sigma > \eta$
θ_{Lu}	$\dfrac{1}{1+\frac{\alpha}{1-\alpha}\left[\left(\frac{AK}{B_uL_u}\right)^a\left(\frac{B_sL_s}{B_uL_u}\right)^{1-a}\right]^{\frac{\sigma-1}{\sigma}}}$	$(1-\alpha)\left(\frac{B_uL_u}{Y}\right)^{\frac{\sigma-1}{\sigma}} =$ $= \dfrac{1}{1+\frac{\alpha}{1-\alpha}\left[a\left(\frac{AK}{B_uL_u}\right)^\rho+(1-a)\left(\frac{B_sL_s}{B_uL_u}\right)^\rho\right]^{\frac{\sigma-1}{\sigma\rho}}}$
θ_X	$1-\dfrac{1}{1+\frac{\alpha}{1-\alpha}\left[\left(\frac{AK}{B_uL_u}\right)^a\left(\frac{B_sL_s}{B_uL_u}\right)^{1-a}\right]^{\frac{\sigma-1}{\sigma}}}$	$\alpha\left(\frac{X}{Y}\right)^{\frac{\sigma-1}{\sigma}} =$ $=1-\left(\dfrac{1}{1+\left(\frac{\alpha}{1-\alpha}\left[a\left(\frac{AK}{B_uL_u}\right)^\rho+(1-a)\left(\frac{B_sL_s}{B_uL_u}\right)^\rho\right]^{\frac{\sigma-1}{\sigma\rho}}\right)}\right)$
θ_L	$1-\dfrac{a}{1+\frac{1-\alpha}{\alpha}\left(\frac{B_uL_u}{B_sL_s}\right)^{\frac{\sigma-1}{\sigma}}}$	$1-a\,(Ak)^\rho\,\alpha\left(\frac{Y}{X}\right)^{\frac{\sigma(\rho-1)+1}{\sigma}} =$ $=1-a\,(Ak)^\rho\,\alpha^{\frac{\sigma\rho}{\sigma-1}}\left\{1+\frac{1-\alpha}{\alpha}\left(\frac{B_uL_u}{B_sL_s}\right)^{\frac{\sigma-1}{\sigma}}\left[a\left(\frac{AK}{B_sL_s}\right)^\rho+(1-a)\right]^{\frac{1-\sigma}{\sigma\rho}}\right\}^{\frac{\sigma(\rho-1)+1}{\sigma-1}}$ where $(Ak)^\rho = \dfrac{1}{\left[\alpha\left(a+(1-a)\left(\frac{B_sL_s}{AK}\right)^\rho\right)^{\frac{\sigma-1}{\sigma\rho}}+(1-\alpha)\left(\frac{B_uL_u}{AK}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma\rho}{\sigma-1}}}$
θ_K	$\dfrac{a}{1+\frac{1-\alpha}{\alpha}\left(\frac{B_uL_u}{B_sL_s}\right)^{\frac{\sigma-1}{\sigma}}}$	$a\,(Ak)^\rho\,\alpha\left(\frac{Y}{X}\right)^{\frac{\sigma(\rho-1)+1}{\sigma}} =$ $=a\,(Ak)^\rho\,\alpha^{\frac{\sigma\rho}{\sigma-1}}\left\{1+\frac{1-\alpha}{\alpha}\left(\frac{B_uL_u}{B_sL_s}\right)^{\frac{\sigma-1}{\sigma}}\left[a\left(\frac{AK}{B_sL_s}\right)^\rho+(1-a)\right]^{\frac{1-\sigma}{\sigma\rho}}\right\}^{\frac{\sigma(\rho-1)+1}{\sigma-1}}$ where $(Ak)^\rho = \dfrac{1}{\left[\alpha\left(a+(1-a)\left(\frac{B_sL_s}{AK}\right)^\rho\right)^{\frac{\sigma-1}{\sigma\rho}}+(1-\alpha)\left(\frac{B_uL_u}{AK}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma\rho}{\sigma-1}}}$
θ_{Ls}	$1-\dfrac{a}{1+\frac{1-\alpha}{\alpha}\left(\frac{B_uL_u}{B_sL_s}\right)^{\frac{\sigma-1}{\sigma}}}-$ $\dfrac{1}{1+\frac{\alpha}{1-\alpha}\left[\left(\frac{AK}{B_uL_u}\right)^a\left(\frac{B_sL_s}{B_uL_u}\right)^{1-a}\right]^{\frac{\sigma-1}{\sigma}}}$	$\theta_L-\theta_{Lu}= \theta_X-\theta_K$

Note: Production function is $Y = \left\{\alpha\left[(AK)^a\,(B_sL_s)^{1-a}\right]^{\frac{\sigma-1}{\sigma}}+(1-\alpha)\,(B_uL_u)^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}}$ in Case 3,

and $Y = \left\{\alpha\left[a\,(AK)^\rho+(1-a)\,(B_sL_s)^\rho\right]^{\frac{\sigma-1}{\sigma\rho}}+(1-\alpha)\,(B_uL_u)^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}}$ in Case 4.

$$LS_{PC,I} = 1 - \Omega a \left(Ak \right)^{\rho} \alpha^{\frac{\sigma\rho}{\sigma-1}} \left\{ 1 + \frac{1-\alpha}{\alpha} l^{\frac{1-\sigma}{\sigma}} \left[a \left(\frac{AK}{B_s L_s} \right)^{\rho} + (1-a) \right]^{\frac{1-\sigma}{\sigma\rho}} \right\}^{\frac{\sigma(\rho-1)+1}{\sigma-1}} \tag{13}$$

where $\Omega = \dfrac{\gamma^{\frac{\lambda}{\lambda-1}}}{\left[1 - \frac{(1-\gamma)^{\lambda}}{\left(\frac{p_I}{p} \right)^{\lambda-1}} \right]^{\frac{1}{\lambda-1}}}$ and $LS_{PC,I}$ denotes the labour share under perfect com-

petition and a production function that incorporates intermediate inputs. To interpret the effect of Ω on the labour share, it is convenient to bear in mind that equilibrium value added can be expressed as a function of Ω and the CES composite of primary inputs (see Appendix 3):

$$Y = \Omega \left[\alpha X^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \left(B_u L_u \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{14}$$

Expression (14) defines the value added in the presence of intermediate inputs. One can see that if $\gamma = 1$, then $\Omega = 1$, and the value added and the labour share are respectively given by expressions (5) and (9), i.e. we are back to the case where there are no intermediate inputs (see case 4 in Table 3). By contrast, as $\gamma \to 0$, then $\Omega \to 0$, and the value added gets smaller. Amongst other things, this implies that a policy encouraging the adoption of new energy-saving technologies (i.e. an increase in the value of γ) raises value added. This policy can thus be seen as a win-win strategy for it generates a higher labour share with no detriment to the capital share.

Appendix 3 shows that, everything else being equal, $\frac{\partial LS_{PC,I}}{\partial \left(\frac{p_I}{p} \right)} > 0$. Thus, an increase in the relative price of intermediate inputs raises the labour share unambiguously. This is due to the capital-skill complementarity hypothesis, which implies that there is higher complementarity (or less substitution) between physical capital and intermediate inputs than between aggregate labour and intermediate inputs. This means that, all other things being equal, an increase in the price of intermediate inputs increases the demand for labour relative to capital, raising relative wages and pushing up the labour share.

Market conditions

This section departs from the perfectly competitive framework by assuming that firms operate in a monopolistic product market and that there is bargaining over wages and employment in the labour market. The connection between real wages and the marginal

productivity of labour is therefore broken, which provides additional explanatory power to account for medium-term labour share movements.

IMPERFECT COMPETITION IN THE PRODUCTS MARKET

Under perfect competition, the labour share is the product of the marginal productivity of labour times the inverse of the average productivity of labour, i.e. the share of value added accruing to labour is (technologically) determined by the employment elasticity of output. Imperfect competition in the products market drives a wedge between the marginal product of labour and the real wage given by the mark-up, which is one institutional variable affecting the labour share.

Under imperfect competition, firms set prices over marginal costs in the following way:

$$p = (1 + \mu)\, mc = (1 + \mu)\, \frac{W_i}{Y'_{Li}}, i = u, s \tag{15}$$

where p, MC and μ respectively denote the value added deflator and the firm's marginal costs and mark-up. W_i and Y'_{Li} represent the nominal wage and marginal productivity of each type of labour. Working out the real wage $w_i = \frac{W_i}{p}$ as a function of the mark-up one gets the labour share under imperfect competition:

$$LS_{IC} = \frac{1}{(1 + \mu)} \frac{Y'_{Lu} L_u + Y'_{Ls} L_s}{Y} \tag{16}$$

Under imperfect competition, the labour share is situated below its level in perfect competition. As indicated by equation (16), non-competitive firms are willing to pay a lower level of real wage for any given level of employment, i.e. the labour demand shifts leftwards (in a magnitude that depends on the mark-up) and crosses the labour supply for lower levels of employment and the real wage.

If the production function is given by (11), one may combine (13) with (16) to obtain the following expression for the labour share (see Appendix 2):

$$LS_{IC,I} = \frac{1}{(1 + \mu)} \left(1 - \Omega a\, (Ak)^\rho\, \alpha^{\frac{\sigma\rho}{\sigma-1}} \left\{ 1 + \frac{1-\alpha}{\alpha} l^{\frac{1-\sigma}{\sigma}} \left[a \left(\frac{AK}{B_s L_s} \right)^\rho + (1-a) \right]^{\frac{1-\sigma}{\sigma\rho}} \right\}^{\frac{\sigma(\rho-1)+1}{\sigma-1}} \right) \tag{17}$$

where $LS_{IC,I}$ denotes the labour share under imperfect competition and a production function that incorporates intermediate inputs. Equation (17) indicates that, ceteris

paribus, the labour share is a decreasing function of the mark-up.

BARGAINING IN THE LABOUR MARKET

Bargaining in the labour market leads to a different pattern of real wages and employment than under perfect competition. Bargaining can take place along two dimensions, according to whether the firm retains the right to manage employment. In the "right-to-manage" model, the firm and the union first bargain over the real wage, and then employment is chosen by the firm unilaterally so as to maximize profits (i.e. the firm chooses a point on the labour demand curve). As pointed out by Leontief (1947), this solution is not efficient in that either the union and/or the firm could be made better off by bargaining over employment as well as wages (the so-called "efficient bargaining").

It is often argued that wages and employment are not determined in the same way in the skilled and unskilled labour markets. In this work, we assume that the market of skilled labour is perfectly competitive, as alternative ways of modelling it (including, e.g., the efficiency wage model or the search paradigm) would introduce greater complexity without affecting the basic results. By contrast, the unskilled wage and employment levels are supposed to be determined in an efficient bargaining fashion. This modelling choice is driven by two considerations. First, unskilled workers are more likely to be covered by union agreements than skilled workers, since they represent the bulk of unions' membership (see Acemoglu *et al.* 2001). Second, our choice is motivated by the need to have a framework in which the unskilled workers' bargaining power, their wages and employment levels all move in the same direction over time. The past three decades have been characterised by a decline in the share of unskilled labour, a fall in their wages relative to skilled and, most likely, a reduction in their bargaining power. Efficient bargaining delivers this result, because of the positive slope of the contract curve[12][13].

Under efficient bargaining for unskilled labour, the union and the firm jointly determine W_u and L_u. This corresponds to the solution of the following Nash bargaining problem[14]:

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[12]   As Blanchard and Giavazzi (2003) put it: "Why assume efficient bargaining? First, it seems like a natural assumption in this context. But also, we want to capture the possibility that firms may not be operating on their demand for labour. In more informal terms, we want to allow for the fact that, when there are rents, stronger workers may be able to obtain a higher wage without suffering a decrease in employment, at least in the short run. Efficient bargaining naturally delivers that implication".

[13]   Note also that modelling wage bargaining for the unskilled according to a right-to-manage paradigm would leave unaffected any of the expressions for the labour share discussed so far. This is because, like in the competitive labour market case, the equilibrium position for real wages and employment under the right-to-manage approach still lies on the labour demand, so, in that regard, changes in the bargaining power of workers affect the labour share thorough variations in the {real wage, employment} equilibrium along the labour demand curve (see Bentolila and Saint Paul, 2003 p. 14 for details).

[14]   It is worthwhile noting that the net gain to the firm from reaching the bargain is defined in (18) in terms of nominal output and nominal wages. This is because in (18) one derives not only with respect to wages, but also

$$\max_{W_u, L_u} \left\{ \frac{L_u}{T_u} \left[U\left(W_u\right) - U\left(RW_u\right) \right] \right\}^{\beta} \left\{ P\left(Y\right) Y\left(\overline{K}, \overline{L_s}, L_u\right) - W_u L_u - \overline{W_s L_s} \right\}^{1-\beta} \tag{18}$$

where $L_u, T_u, U(.), W_u, RW_u, P(Y), Y,$ and β all refer to unskilled workers and respectively stand for employed union members, total union members, the representative worker utility, nominal wages, the reservation wage, the inverse of the demand curve faced by any (imperfectly competitive) firm, the firm's value added and the union's bargaining power. The bar symbol over a variable is used to indicate that this is taken as given when negotiating over W_u and L_u. Note that the firm's surplus is defined in such a way that if the union has all the bargaining power, unskilled workers will absorb any surplus that remains after remunerating the skilled by their marginal productivity, i.e. the labour share is equal to 1 under $\beta = 1$.

The equilibrium is characterised by the following two equations:

$$\frac{Y'_{L_u}}{(1+\mu)} - w_{u,IC,EB} = -\frac{\left[U\left(w_{u,IC,EB}\right) - U\left(RW_u\right) \right]}{U'\left(w_{u,IC,EB}\right)} \tag{19}$$

$$\left(\frac{W_u}{P} \right)_{IC,EB} = w_{u,IC,EB} = \beta \left(\frac{Y - \overline{w_s L_s}}{L_u} \right) + (1-\beta) \frac{Y'_{L_u}}{(1+\mu)} \tag{20}$$

where $\overline{w_s}, w_{u,IC,EB}$ respectively stand for skilled workers' real wages (implied by expression (15)) and unskilled workers' real wages under efficient bargaining. Equation (19) is the "contract curve". It states that an efficient wage and employment outcome is one where the slopes of the isoprofit curve and the indifference curve are the same[15]. The contract curve starts at the competitive equilibrium (under $w_{u,IC,EB} = RW_u$) and generally lies to the right of the labour demand curve (for any $w_{u,IC,EB} > RW_u$). This implies that the value of the marginal product of labour is generally less than the real wage by an amount which is equal to the union marginal rate of substitution of employment for wages. It can also be shown that the contract curve is upward sloping. Intuitively, as wages are increased above the competitive level ($\beta > 0$), any members who are laid off have an increasing opportunity cost of being unemployed. The union therefore insures members

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with respect to employment. As an imperfectly competitive firm faces a downward-sloping curve, one is then obliged to consider the reduction in prices arising from a marginal increase in employment.

[15] Efficiency means that the marginal rates of substitution of employment for wages, for both the union and the firm, are equal.

Figure 3: *Labour market outcomes for unskilled labour under efficient bargaining.*
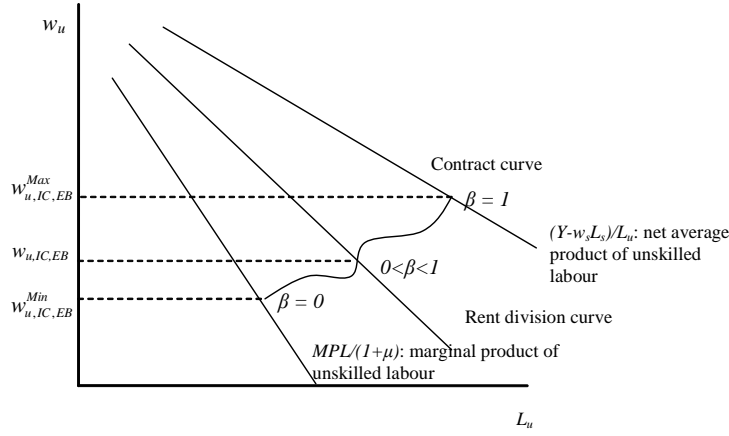
against this risk by bargaining for increased employment[16].

The specific point of the contract curve chosen depends on the relative bargaining power of the firm and of the union, as indicated by the "rent division curve" (20). If the union has no power ($\beta = 0$), the rent division curve collapses to the marginal product of labour (corrected for the mark-up), i.e. $\left(\frac{W_u}{P}\right)_{IC,EB} = \frac{Y'_{L_u}}{(1+\mu)}$, the outcome under perfect competition. If the firm has no power ($\beta = 1$), the rent division curve becomes the average product of labour net of the skilled workers' wage bill, i.e. $\left(\frac{W_u}{P}\right)_{IC,EB} = \frac{Y - \overline{w_s}\overline{L_s}}{L_u}$. The equilibrium outcome is illustrated in Figure 3, where the efficient wage and employment levels are given by the intersection of the rent division curve and the contract curve.

Substituting the real wage according to (20) into the definition of the labour share given by (16), one gets:

$$LS_{IC,EB} = \beta + \left(\frac{1-\beta}{1+\mu}\right)\left[\frac{Y'_{L_u}L_u + Y'_{L_s}L_s}{Y}\right] \tag{21}$$

By comparing equations (16) and (21) it can be seen that, under efficient bargaining

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[15] For a formal proof see, for instance, Booth (1995), p. 130.

[16] This reasoning assumes that union's members are risk-averse, that is $U'(w) > 0$ and $U''(w) > 0$, which is the relevant empirical case for unskilled workers. If members were risk-neutral, the contract curve would be vertical; members are not offered insurance against the risk of being unemployed. If members were risk-loving, the contract curve would be negatively sloped.

in the labour market, the labour share will be generally higher when compared to a situation where no bargaining power is allocated to unskilled workers. The labour share given by (21) ranges between its value provided by equation (16) (if $\beta = 0$, i.e. all bargaining power is allocated to firms) and 1 (if $\beta = 1$, i.e. all bargaining power rests with unskilled workers).

If the production function for gross output is given by (11), then one can get the following expression for the labour share:

$$LS_{IC,I,EB} = \beta + \left(\frac{1-\beta}{1+\mu}\right) \left(\frac{1 - \Omega a \, (Ak)^\rho \, \alpha^{\frac{\sigma\rho}{\sigma-1}}}{\left\{ 1 + \frac{1-\alpha}{\alpha} l^{\frac{1-\sigma}{\sigma}} \left[a \left(\frac{AK}{B_s L_s}\right)^\rho + (1-a) \right]^{\frac{1-\sigma}{\sigma\rho}} \right\}^{\frac{\sigma(\rho-1)+1}{\sigma-1}}} \right) \quad (22)$$

where $LS_{IC,I,EB}$ is the labour share with firms operating under imperfect competition, varying price of intermediate inputs, perfect competition in the skilled labour market and efficient bargaining in the unskilled labour market. From (22), one can obtain the partial derivative of the labour share with respect to β, which is equal to:

$$\frac{\partial LS_{IC,I,EB}}{\partial \beta} = 1 - \frac{1}{1+\mu} \left(\frac{1 - \Omega a \, (Ak)^\rho \, \alpha^{\frac{\sigma\rho}{\sigma-1}}}{\left\{ 1 + \frac{1-\alpha}{\alpha} l^{\frac{1-\sigma}{\sigma}} \left[a \left(\frac{AK}{B_s L_s}\right)^\rho + (1-a) \right]^{\frac{1-\sigma}{\sigma\rho}} \right\}^{\frac{\sigma(\rho-1)+1}{\sigma-1}}} \right)$$

Given that $\mu > 0$ and that the term in brackets is strictly below 1, it always follows that $\frac{\partial LS_{IC,I,EB}}{\partial \beta} > 0$. The rationale behind this result is as follows. In the right-to-manage model, a higher level of β would imply a decline in the demand for unskilled workers (increasing with sigma) as firms substitute human and physical capital for unskilled labour. However bargaining solely over wages is Pareto-inefficient. With efficient bargaining and risk adverse individuals, the slope of the contract curve is positive, i.e. the efficient allocation from the point of view of the bargainers implies that employment is rising with wages. This is because as wages are increased above the competitive level, those that are laid off have an increasing opportunity cost of being unemployed (i.e. the newly unemployed cannot enjoy higher negotiated wages). Thus, the union insures against this risk by bargaining for increased employment. The tougher unions representing unskilled workers are at the bargainnig table (i.e. the higher β) the lower the increase in the wages and the higher the increase in employment (i.e. the contract curve defined in the {real wage, employment} space flattens as β increases). This is associated with a higher labour share.

ADJUSTMENT COSTS

This section explores the implications for the labour share of labour hoarding. Labour hoarding is to a large extent determined by adjustment costs (such as firing and hiring restrictions, search and training costs), which affect the behaviour of employment in two ways:

- Hiring and firing restrictions reduce the fluctuations of employment, as they induce less hiring when demand is expanding and less firing when it is contracting.
- If adjustment costs are convex in the change in employment, they result in a gradual distribution over time of any given magnitude of the change in employment, which rationalizes a lagged response of employment to changes in output.

This means that, even if the technology is Cobb-Douglas, the labour share will fluctuate along the business cycle (see Kessing, 2001). With a more general CES production function, adjustment costs ought to exacerbate labour share's fluctuations, as variations in labour productivity will be more pronounced in presence of labour hoarding.

For ease of analysis, adjustment costs have most often been represented using a convex symmetric function. However, a growing body of empirical literature has rejected the hypothesis of symmetric adjustment in favour of some form of asymmetry (see a comprehensive review of the literature in Hamermesh and Pfann, 1996, Table 1, p. 1280). This has led Pfann and Palm (1993) to assume a general specification for adjustment costs of the form:

$$AC\left(\triangle L\right) = -1 + e^{\phi\triangle L} - \phi\triangle L + \frac{\chi}{2}\left(\triangle L\right)^2 \text{ with } \phi \text{ unrestricted and } \chi > 0 \quad (23)$$

Parameters χ and ϕ characterize the adjustment cost function (23). $\chi > 0$ implies that adjustment costs are a convex function of the quantity of labour, i.e. $AC'\left(\triangle L\right) > 0, AC''\left(\triangle L\right) > 0$. ϕ defines the type of asymmetric adjustment with symmetry implied by $\phi = 0$ and $\phi > 0(\phi < 0)$ representing a situation where the marginal cost of an increase in employment is greater (smaller) than that of a reduction.

In the presence of adjustment costs for both skilled and unskilled employment, the equilibrium in the labour market is characterized by the following expressions:

$$\left(\frac{W_S}{P}\right)_{IC,ac} = w_{s,IC,AC} = \frac{Y'_{L_s} - AC'\left(\triangle L_s\right)}{(1+\mu)} \quad (24)$$

$$\frac{Y'_{L_u} - AC'\left(\triangle L_u\right)}{(1+\mu)} - w_{u,IC,EB,AC} = -\frac{\left[U\left(w_{u,IC,EB,AC}\right) - U\left(RW_u\right)\right]}{U'\left(w_{u,IC,EB,AC}\right)} \quad (25)$$

$$\left(\frac{W_u}{P}\right)_{IC,EB,AC} = w_{u,IC,EB,AC} = \beta \left(\frac{Y - \overline{w_{s,IC,AC}}\overline{L}_s}{L_u}\right) + (1-\beta)\frac{Y'_{L_u} - AC'\left(\triangle L_u\right)}{(1+\mu)} \quad (26)$$

where $w_{s,IC,AC}$ and $w_{u,IC,EB,AC}$ are the real wages for skilled and unskilled workers in the presence of adjustment costs, imperfect competition in the products market and efficient bargaining in the unskilled labour market. The bar symbol over $w_{s,IC,AC}$ in expression (26) indicates that the skilled workers' real wage (implied by expression (24)) is taken as given when bargaining. Expressions (24), (25) and (26) only differ from the original versions (15), (19) and (20) in the marginal costs of adjustment. The modified contract curve (25) implies that the firm will be more reluctant to hire new workers during expansions in the presence of adjustment costs. The new solution for the real wage (26) shows that the firm's threat point (i.e. wages under $\beta = 0$) is given by the marginal product of unskilled labour minus its marginal adjustment cost. Thus, in the presence of adjustment costs, the firm will be willing to pay lower wages for any given level of employment.

To proceed further, it is worth noting that only labour-consuming adjustment costs should be included in the labour share[17]. In what follows it is assumed that all adjustment costs use labour[18]. Accordingly, the labour share is augmented with the remuneration to the labour services that facilitate incorporating additional employment into the firm. If the production function for gross output is given by (11), then one can get the following expression for the labour share:

$$LS_{IC,I,EB,AC} = \beta + \left(\frac{1-\beta}{1+\mu}\right) \left(\frac{1 - \Omega a \left(Ak\right)^\rho \alpha^{\frac{\sigma\rho}{\sigma-1}}}{\left\{1 + \frac{1-\alpha}{\alpha}l^{\frac{1-\sigma}{\sigma}}\left[a\left(\frac{AK}{B_sL_s}\right)^\rho + (1-a)\right]^{\frac{1-\sigma}{\sigma\rho}}\right\}^{\frac{\sigma(\rho-1)+1}{\sigma-1}}}{\frac{1}{(1-\beta)}\frac{AC'(\triangle L_s)L_s}{Y} - \frac{AC'(\triangle L_u)L_u}{Y}}\right)$$
$$+ \frac{AC\left(\triangle L_s\right) + AC\left(\triangle L_u\right)}{Y} \quad (27)$$

Adjustment costs have two effects on the labour share. First, adjustment costs influence firms' demand for labour and the equilibrium real wage (see eq. 24, 25 and 26). For the given average employment level, the firm's marginal costs are higher when the

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[17]   One may distinguish adjustment costs that consume labour or that take the form of a firm's payments to the worker (e.g. severance payments, training costs, recruitment services provided by an employment agency) from costs related neither directly nor indirectly to labour.

[18]   On empirical grounds adjustment costs are a negligible share of income. Thus, including average adjustment costs in expression (27) eventually makes no difference to the level of the labour share and its dynamics.

employment adjustment is costly, which implies a lower wage share. Second, labour income comprises income from work and a monetary transfer corresponding to the value of the insurance against unstable employment provided by the adjustment costs. Thus adjustment costs raise the labour share as long as they are equivalent to such payments to workers.

4.2 Dynamic model

In this section we investigate the dynamic behaviour of the labour share within the Solow growth model. Let us consider the following two-level CES production technology:

$$Y_t = B_t \left[\alpha \left(\left[a \left(K_t \right)^{\frac{\eta-1}{\eta}} + (1-a) \left(L_s \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \left(B_u L_u \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{28}$$

and the associated labour share:

$$LS_{PC,t} = 1 - a \left(k_t \right)^{\rho} \alpha^{\frac{\sigma\rho}{\sigma-1}} \left\{ 1 + \frac{1-\alpha}{\alpha} l_t^{\frac{1-\sigma}{\sigma}} \left[a \left(\frac{K_t}{L_{s,t}} \right)^{\rho} + (1-a) \right]^{\frac{1-\sigma}{\sigma\rho}} \right\}^{\frac{\sigma(\rho-1)+1}{\sigma-1}} \tag{29}$$

The only difference between equations (28) and (29) and (5) and (9) is in the specification of technical progress, which is now assumed to be purely labour-augmenting (i.e. Harrod neutral). This is needed for the neoclassical growth model to deliver a steady state solution, thus consistent with the observation that shares of value added accruing to labour show no secular trend.

Let s, δ, n be the (exogenously given) savings rate, the depreciation rate and the population growth rate, respectively. Technical progress is Harrod-neutral and grows at the exogenous rate g. In transitional dynamics, skilled labour is assumed to grow faster than total population $n_{u,t}$, which yields an increasing fraction of skilled labour over time, $(1 - u_t)$.

The dynamics of the labour share is determined by the behaviour of the key ratios $k_t = \frac{K_t}{Y_t}, l_t = \frac{L_{s,t}}{L_{u,t}}$ and $\frac{K_t}{L_{s,t}}$ as given by equations (30) to (33):

$$l_t = \frac{L_{s,t}}{L_{u,t}} = \frac{\frac{L_{s,t}}{N_t}}{\frac{L_{u,t}}{N_t}} = \frac{1 - u_t}{u_t} \tag{30}$$

$$u_t = u_0 e^{t * n_{u,t}} \text{ where } n_{u,t} < n \text{ and } lim_{t \to \infty} n_{u,t} = n \tag{31}$$

$$\frac{K_t}{L_{s,t}} = \frac{\frac{K_t}{N_t}}{\frac{L_{s,t}}{N_t}} = \frac{(1+g) \left[s \frac{Y_{t-1}}{N_{t-1}} + (1-n-\delta-g) \frac{K_{t-1}}{N_{t-1}} \right]}{1-u_t} \tag{32}$$

$$k_t = \frac{K_t}{Y_t} = \frac{\frac{K_t}{N_t}}{\frac{Y_t}{N_t}} = \frac{(1+g) \left[s \frac{Y_{t-1}}{N_{t-1}} + (1-n-\delta-g) \frac{K_{t-1}}{N_{t-1}} \right]}{B \left[\alpha \left[a \left(\frac{K_t}{N_t} \right)^\rho + (1-a)(1-u_t)^\rho \right]^{\frac{\sigma-1}{\sigma\rho}} + (1-\alpha) u_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}} \tag{33}$$

The steady state of the key ratios, denoted with (*), is defined by the following conditions:

$$l^* = \left(\frac{L_s}{L_u} \right)^* = \frac{\left(\frac{L_s}{N} \right)^*}{\left(\frac{L_u}{N} \right)^*} = \frac{1-u^*}{u^*} \tag{34}$$

$$\left(\frac{K}{L_s} \right)^* = \frac{\left(\frac{K}{N} \right)^*}{\left(\frac{L_s}{N} \right)^*} \tag{35}$$

$$sB \left[\alpha \left[a \left(\left(\frac{K}{N} \right)^* \right)^\rho + (1-a)(1-u^*)^\rho \right]^{\frac{\sigma-1}{\sigma\rho}} + (1-\alpha)(u^*)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$
$$= (n+\delta+g) \left(\frac{K}{N} \right)^* \tag{36}$$

$$\left(\frac{L_s}{N} \right)^* = 1-u^* \tag{37}$$

$$k^* = \left(\frac{K}{Y} \right)^* = \frac{\left(\frac{K}{N} \right)^*}{\left(\frac{Y}{N} \right)^*} = \frac{s}{n+\delta+g} \tag{38}$$

Papageorgiou and Saam (2005) establish the existence and stability conditions in the Solow model within the framework of a two-level CES like (28). They show that when both σ and η are positive, a unique and stable steady state $\left(\frac{K}{N}\right)^* > 0$ exists if and only if $B\alpha^{\frac{\sigma}{\sigma-1}} a^{\frac{\eta}{\eta-1}} < \frac{n+\delta}{s}$.

5. Model calibration and simulation

5.1 Model calibration

For calibration purposes we build on the static version described under Section 4.1. Calibration is country-specific, in that both data and parameter values vary depending on the country. Further, the choice of the best specification for the labour share (among the several versions discussed under Section 4.1) may differ across countries.

The model is calibrated to nine EU15 Member States, namely Austria, Belgium, Germany, Denmark, Spain, Finland, France, Italy, and the United Kingdom. The calibration matches average values for 1970-2004[19]. The key ratios $k_t = \frac{K_t}{Y_t}, l_t = \frac{L_{s,t}}{L_{u,t}}$ and $\frac{K_t}{L_{s,t}}$ rely on the following sources. Capital stock corresponds to the variable "real fixed capital stock at 1995 prices, all assets" in the EU KLEMS database, except for Belgium, where capital is taken from the OECD STAN database. The remaining variables are EU KLEMS series. Value added is "gross value added at current basic prices", skilled and unskilled labour are respectively made equal to high-skilled and medium- plus low-skilled workers as defined in the EU KLEMS database. For each level of qualification, the number of hours worked is the product of the share in total hours worked per each skill category and total hours worked by persons engaged. Hours worked per skill category are then divided by hours worked per employee to obtain labour in headcounts.

The paths for capital and labour-augmenting technical progress are consistent with Klump *et al.* (2007) (see Table 4 in p.108 of the referred paper), which is one recent work attempting to quantify biased technological progress in the euro area over the period 1970-2003, i.e. the same period and country coverage adopted in this paper. Their estimates imply that labour-augmenting is higher than capital-augmenting technical progress at any time, that capital-augmenting technical progress vanishes in the limit and that labour-augmenting technical progress approaches around 0.6% in the very long run [20]. For con-

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[19]   The calibration periods, which slightly differ across the countries, are as follows: 1971-2004 for Italy and the UK, 1974-2004 for Finland, 1980-2004 for Belgium, Denmark, Spain, France and Austria, and 1991-2004 for Germany.

[20]   One caveat in our use of such estimates is that we implicitly assume that labour-augmenting technical progress is the same for both skilled and unskilled labour, as technology in Klump *et al.* (2007) is one-level CES in capital and aggregate labour.

venience, the *level* of capital-augmenting technical progress is scaled to match the observed labour share in the first year of the calibration period for each country.

The evolution of mark-ups over time is consistent with the evidence reported in McAdam and Willman (2004b)[21].

Calibration starts by setting the value of the distribution parameters, a, α, and γ. γ matches the weight of value added in gross output. Pinning down α and a requires choosing a value for the elasticity of substitution between the capital-skilled labour composite and unskilled labour, σ. This value has been set for most countries at 1.5, as in Krusell *et al.* (2000). It is then possible to choose α to match the average labour share of the unskilled over the calibration period. Given α, it is straightforward to obtain a, as the sum of all shares equals one.

With the data and values for distribution parameters described above the calibration strategy for each country goes from the most specific (i.e. expression (9)) to the most general case (i.e. expression (27)). This way, it is possible to determine the main drivers underlying labour share movements in each country, including technological progress, changes in the relative price of intermediate inputs, mark-ups, the unskilled workers' bargaining power and adjustment costs in employment. For each country, the values for the technological parameters σ, η and λ, and for the parameters defining the adjustment cost function φ and χ are in line with the existing empirical evidence. Specifically, the calibrated values of η fall within the range of values surveyed in Hamermesh (1993a) (chapter 3). Chosen values for λ closely follow those estimated by Saito (2004). The parameters φ and χ are consistent with those predicated by the literature on asymmetric adjustment costs in employment, as argued further below.

Leaving aside country-specific features, there seems to be a common story to the explanation of medium-term labour share movements in European countries (Figure 4). The driving force behind the observed downward trend is technology. Given the characteristics of the production function, there are various channels through which the labour share is affected along the transitional dynamics (i.e. the medium term). These channels are as follows:

- with capital-skill complementarity ($\sigma > \eta$), capital-augmenting technical progress pushes the labour share downwards;
- with high substitution between skilled and unskilled labour ($\sigma > 1$), the labour share responds negatively to the historical increase in the relative supply of skilled labour;

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[21]   Their theoretical framework contains a multi-sector model of imperfect competition. By allowing price and income elasticities to differ across sectors, the aggregation of the profit maximisation implies that the aggregate mark-up increases over time due to sectoral shifts in the economy, even though the mark-up in each sector remains constant. Specifically, the upward trend in the mark-up reflects an increasing weight of sectors with high mark-ups (e.g. services) to the detriment of those with low mark-ups (e.g. manufacturing).

- with high substitution between capital and unskilled labour ($\sigma > 1$), the labour share responds negatively to the historical increase in the capital to unskilled labour ratio;
- with $\eta > 0$ the labour share responds negatively to the historical fall in the capital to skilled labour ratio[22].

The only two countries that do not follow this general pattern are the United Kingdom and Denmark. The United Kingdom conforms fairly well with the Cobb-Douglas case, i.e. $\sigma = \eta = 1$, which implies a constant labour share over the long run (see Case 1 in Table 3). Calibrated parameters for Denmark adopt a Cobb-Douglas technology for the capital-skilled labour composite ($\eta = 1$) while keeping a value of 1.5 for σ. The intuition of why these values imply a milder decline in the labour share is provided in Section 5.2 below, with reference to the simulation exercise conducted with the dynamic version of the model.

Note that while the value σ has been set for most countries at 1.5, the value of η is country-specific. The value of η *relative to* σ, i.e. the extent to which capital is more substitutive to unskilled than skilled labour, should reflect country-specific technological features. In the literature, the aggregate elasticity of substitution has been broadly defined as a measure of the efficiency of the productive system (e.g. de La Grandville, 1989). In the vein of Hicks (1963), the value of η should capture the ease of inter-sectoral factors' reallocation, which strongly depend on the sectoral elasticities of substitution. In an attempt to investigate this hypothesis, we have used EU KLEMS data from 18 OECD countries[23] covering the period 1970-2004 disaggregated by 9 main market industries[24] to estimate the relationship between the labour share and the capital-output ratio given by (9). We estimate a fixed-effect model which allows for country, industry and time specific fixed effects. The econometric evidence suggests that the capital-output ratios are significant, an indication that technological differences are a source of labour share movements. According to these estimates, capital and skilled labour appear as highly substitutive ($\eta > 1$) in Agriculture, Electricity and Finance, and low substitutes ($\eta < 1$) in Manufacturing, Transport and Communications, and Hotels and Restaurants. A unitary elasticity of substitution ($\eta = 1$, i.e. Cobb-Douglas technology) seems to prevail in Construction and Trade. Cal-

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[22]   There are nevertheless a few countries (i.e., Belgium, Germany and Finland) where the capital-skilled labour ratio has increased over time.

[23]   The sample includes Austria, Belgium, the Czech Republic, Germany, Denmark, Spain, Finland, France, Hungary, Italy, Luxembourg, the Netherlands, Poland, Sweden, the United Kingdom, the United States, Canada and Japan.

[24]   The sectoral breakdown adopts the one-digit level of the NACE classification, which includes 9 broadly-defined industries, namely, Agriculture, Hunting, Forestry and Fishing (A-B), Mining and Quarrying (C), Total Manufacturing (D), Electricity, Gas and Water Supply (E), Construction (F), Wholesale and Retail Trade (G), Hotels and Restaurants (H), Transport and Storage and Communication (I), Finance, Insurance, Real Estate and Business Services (J-K).

culating the weight in total value added[25] of the sectors with $\eta \geq 1$ yields the following ranking of countries (from higher to lower weight of sectors with $\eta \geq 1$): the United Kingdom, Denmark, France, Belgium, Italy, Germany, Spain, Austria and Finland. Although this ranking generally supports the country-specific values chosen for η as summarised by Table 4, it does not admittedly explain the case of France (which ranks high in terms of sectoral weights but is calibrated at $\eta = 0.7$) and Germany (which occupies an intermediate to low position in terms of sectoral weights but is calibrated at $\eta = 1.3$).

In an attempt to go beyond the technological determinants of inter-sectoral factors' substitution, Klump and Preissler (2000) emphasize the role of institutional factors. It is argued that high levels of efficiency and the aggregate elasticity of substitution are associated with competitive labour markets, a high degree of openness and the presence of institutions that favour the transfer of ideas among individuals and nations[26]. Thus, in addition to the sectoral dimension discussed above, institutional arrangements should contribute to rationalize the values for η reflected in Table 4. For instance, Spain is one country characterized by a relatively low degree of market competition, comparatively low degree of openness and poor innovation performance, which would all justify a low value for η. Conversely, Denmark's high value for η could be justified in light of high levels of competition, openness and innovation.

Whereas the interaction between capital-augmenting technical progress, changes in relative factor quantities and the elasticities of substitution play a key role in explaining trends, the incorporation of intermediate inputs has the virtue of accommodating medium-term swings, with changes in their relative price leading to a temporary change in the same direction in the labour share.

Adjustment costs improve the fit of short- to medium-term labour share movements in all countries except Austria and Belgium[27], with labour hoarding being a major cause of the relatively dampened and lagged response of employment to output. Empirical literature on adjustment costs for specific countries is scant [28]. The few studies available in

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[25] This calculation is made on the basis of averages over the past decade.

[26] Openness is thoroughly discussed in Ventura (1997), who has shown that a small country open to international trade can be modelled as possessing a linear aggregate production function. More generally, globalisation is claimed to increase the elasticity of labour demand with respect to the real wage (see OECD, 2007). On the other hand, Weder and Grubel (1993) claim that industry-wide research associations can also cause high elasticities of substitution, as they favour knowledge spillovers which result in new methods of production.

[27] The best fit for Austria is obtained upon calibration of expression (8) (only technology). The preferred specification for Belgium is expression (13) (production function extended with intermediate inputs).

[28] Hamermesh (1993a) summarises the stylised facts on employment dynamics: 1) The lag in adjusting employment demand is fairly short, with a half-life of perhaps three to six months; 2) Hours per worker are adjusted more rapidly than employment, implying that costs of adjusting them are less than those of changing employment levels. Taken together this literature implies that adjustment costs for labour are not large. The few studies that have tried to estimate their size directly on aggregate data confirm this conclusion, for they imply that the

the literatue[29][30] point to the following classification of countries:

- Asymmetric adjustment costs with $\phi > 0$ (i.e. the marginal cost of an increase in employment is greater than that of a reduction) can be postulated for the UK and Spain. This is consistent with the evidence found by Messina and Vallanti (2006) who identify these two countries as the only cases among EU15 economies where job reallocation (i.e. the sum of job creation and job destruction) is counter-cyclical[31]. Blanchard and Wolfers (2000) rationalize the Spanish case with reference to the development of fixed-term contracts rather than the weakening of the protection of indefinite workers.

- Asymmetric adjustment costs with $\phi < 0$ (i.e. the marginal cost of an increase in employment is lower than that of a reduction) seem to prevail in Italy and France. Empirical works providing support for a slightly pro-cyclical pattern of job reallocation in these two countries are Jaramillo *et al.* (1993) for Italy, Lagarde *et al.* (1994) for France and Bentolila and Bertola (1990) for both Italy and France.

- Symmetric adjustment costs in employment ($\phi = 0$) appear as a reasonable assumption in the case of Germany, Denmark and Finland. Acyclical pattern in job reallocation is found by Boeri and Cramer (1992) for Germany, and Messina and Vallanti (2006) for Denmark and Finland.

The evolution of mark-ups over time helps explain declining labour shares in only three countries, i.e. Finland, Spain and Germany. Oliveira *et al.* (1996) find that Finland is the country with the highest increase in mark-ups for manufacturing sectors between the 1970s and the 1980s. Spain and Germany are two countries where the shift in production away from manufacturing to market services has been the most dramatic, thereby providing support to the McAdam and Willman's sectoral shift hypothesis (op.cit). Fi-

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per-period costs are not much more than one percent of the per-period payroll cost. The calibration exercise in this section is consistent with these figures.

[29]   Some of the empirical studies quoted below in support of specific structures of adjustment costs refer to data older than a decade ago. Although this is certainly a caveat of the calibration exercise presented in this section, the assumption that employment protection has remained roughly stable since the 1970s can be adopted without loss of accuracy. See Blanchard and Wolfers (2000) for a discussion on the biggest EU15 economies.

[30]   The studies quoted below are of three kinds: i) based on correlations between job reallocation and the output gap; ii) based on macroeconomic estimations of the adjustment cost function; and iii) based on estimations of the adjustment cost function at the micro level, most often focusing on manufacturing. Another strand of the literature has sought to estimate correlations between job reallocations and the strictness of Employment Protection Legislation (EPL), with varying results depending on the study, which therefore do not allow for a classification of countries. In this line, Garibaldi *et al.* (1997) find a negative correlation between job reallocation and the strictness of EPL. On the contrary, similar correlations in OECD (1999) show a very weak negative association between different indicators of the strictness of EPL and job turnover rates. This obviously stems from the fact that barriers to the layoff of workers are expected to hinder both job creation and destruction, having ambiguous effects on the average level of labour demand.

[31]   Note that although job creation is clearly pro-cyclical and job destruction is counter-cyclical, the volatility of the two flows over the cycle may differ. Estimates for Anglo-Saxon countries and Spain suggest that the increase in job destruction during economic downturns tends to be stronger than the increase in job creation during upturns, resulting in counter-cyclical movements in job reallocation.

Table 4 − Country-Specific Parameter Values

Calibration on the basis of EU KLEMS data.

The period used for calibration slightly differs across countries (see footnote 18).

	AT	BE	DE	DK	ES	FI	FR	IT	UK
LS preferred specification									
Technology	X	X	X	X	X	X	X	X	X
Intermediate inputs		X		X	X	X	X		
Mark-up			X		X	X			
Adjustment costs			X	X	X	X	X	X	X
Elasticities of substitution									
σ	1.5	1.5	1.5	1.5	1.5	1	1.5	1.5	1
η	0.4	0.7	1.3	1	0.3	0.5	0.6	0.7	1
λ	1.5	1.6	1	1.8	0.7	3	1.7	1.3	0.3
Product- and labour-market parameters									
μ			(*)		(*)	(*)			
β	0	0	0	0	0	0	0	0	0
χ			2	2	0.5	2	1.5	1.5	0.5
ϕ			0	0	1	0	-1	-1	1

Source: Commission services.

(*): Evolution of mark-ups over time as in McAdam and Willman (2004b).

nally, the unskilled workers' bargaining power does not provide any further explanatory power to labour share movements. This result is in line with Bentolila and Saint Paul (2003) who, based on econometric procedures, obtain a non-significant coefficient for the variable meant to capture the bargaining power of workers[32].

5.2   Model simulation

The previous section has shown that most of the declining pattern in labour shares in nine EU15 Member States is governed by technological factors, namely, the interaction of capital deepening, capital-augmenting technical progress and labour substitution across skill categories. To illustrate the force of the technological explanation we use the dynamic model developed in Section 4.2 to simulate the impact on the labour share of a permanent reduction in the fraction of unskilled.

We focus on two countries, namely France and Denmark, which received quite similar treatment in our calibration strategy (Table 4). We assume perfectly-competitive markets and thus abstract from any effects induced by changes in the mark-up, the bargaining power or the adjustment costs. For simplicity, we also neglect the influence of changes in the relative price of intermediate inputs. Our main aim is to show that, for a given increase in the share of skilled workers, the reduction in the labour share crucially depends on the

[32]   As claimed by Bentolila and Saint Paul (2003) "(...) the straightforward interpretation would be that wage bargains are not effcient, with the right-to-manage model, say, being a more appropriate description of reality".

Figure 4: *Observed and calibrated labour shares in selected EU15 Member States. Calibration on the basis of EU KLEMS data. The period used for calibration slightly differs across countries* (see footnote 18).
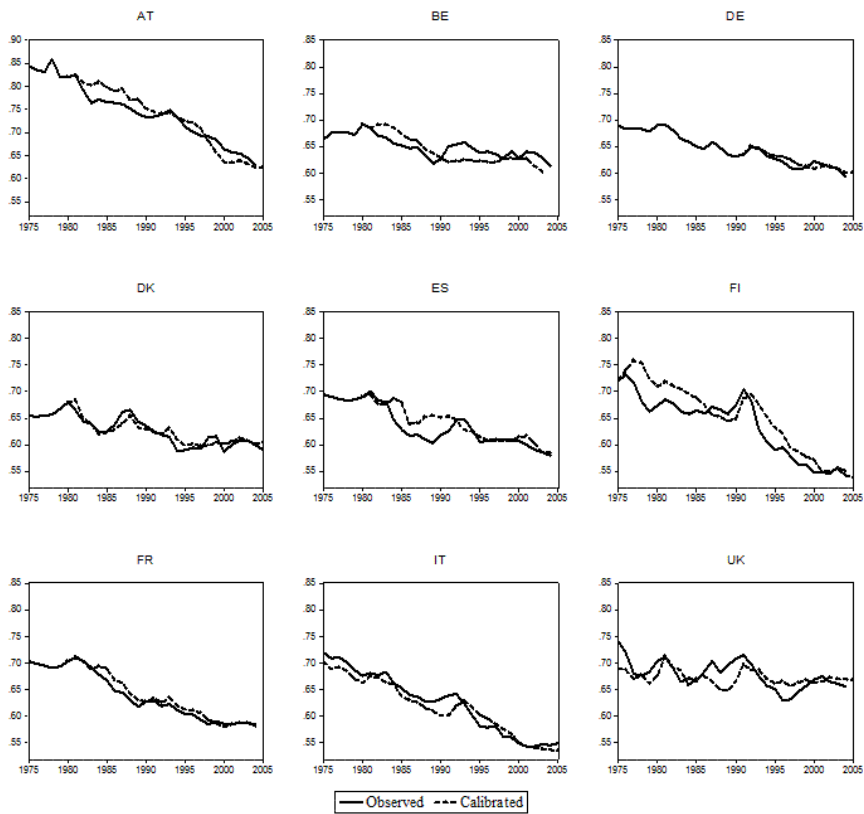
value of η. Note that, among the three input ratios which determine the labour share (see expression (10) and accompanying paragraph), only the relative supply of skilled to unskilled labour can be regarded as genuinely exogenous. Once we decide upon its path, both capital per skilled worker and capital per unskilled worker will result from capital accumulation as standard in the neoclassical growth model.

The starting point of the simulation exercise is 2000, where it is assumed that both economies are in transitional dynamics. Calibration was devised in such a way that the only difference between the two countries is in the value of η, which is set to 1 in Denmark and 0.6 in France (see Table 4)[33].The annual rate of capital depreciation δ is equal to 4%. It is assumed that labour-augmenting technical progress g stabilises at 1.1% in 2015 (starting from around 1% in 1999), while the savings rate s and the population growth rate n only reach their steady state levels in 2060, at respectively 19.3% and 0.1%. A hyperbolic convergence path has been adopted between the initial values of s and n in 2000 and their permanent levels in 2060. In both countries unskilled labour grows at a lower rate than total population in transitional dynamics, i.e. $n_{u,t} < n_t$, which yields a decreasing fraction of unskilled labour over time, u_t. Specifically, the conservative assumption made in the simulated scenario is that there will be a 5% reduction in u_t between 2000 and 2060. The starting value for the *level* of labour-augmenting technical progress is calibrated to match the value of effective income per capita in 2000.

The results are displayed in Figure 5. Given the initial position of both economies, the calibrated values for fundamental parameters and the fraction of unskilled labour, transitional dynamics are characterised by the accumulation of capital, with all the three ratios $\frac{K_t}{N_t}$, $\frac{K_t}{L_{s,t}}$ and $\frac{K_t}{L_{u,t}}$ increasing over time. To interpret the impact on the labour shares recall that, given the properties of the production function, increases in both $\frac{L_{s,t}}{L_{u,t}}$ and $\frac{K_t}{L_{u,t}}$ cause a fall in the labour share; conversely, an increase in $\frac{K_t}{L_{s,t}}$ pushes the labour share upwards over time (see Section 4.1). Our simulations indicate that the former two effects predominate over the latter one, thereby implying a reduction in the labour share along the transition path. As indicated by the stability of the capital-output ratio, the two economies reach the steady state by 2060. By that time, the labour share has declined by more than 10 percentage points in France[34], against only 4 percentage points in Denmark.

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[33]   Chosen values for δ, g, s, and n are consistent with the projection exercise elaborated by the Economic Policy Committee's Ageing Working Group and the European Commission (2009). The Economic Policy Committee (EPC) gathers senior officials from national finance ministries and central banks and serves to prepare the ECOFIN Council. The EPC's Ageing Working Group was established to study the implications of ageing populations for public finances in areas such as pensions, health and education.

[34]   Although the order of magnitude of these results may surprise on the upside, they are strikingly similar to their historical counterparts. Between 1980 and 2000 the share of the unskilled in total employment in France dropped from 0.96 to 0.9, while the labour share fell from 0.70 to 0.58. Thus the percentage point reduction in the labour share more or less doubled the percentage point reduction in the share of the unskilled.

To interpret these results note that, according to expression (10), the labour share can be expressed in terms of the proportion of skilled workers, the wage premium and the unskilled workers' labour share. Given that, by assumption, the p.p. reduction in the fraction of unskilled workers is the same, any difference in labour share behaviour between the two countries must arise from the remaining two variables. On the one hand, an increase in the fraction of skilled labour $\frac{L_{s,t}}{L_{u,t}}$ will create an excess supply of qualified workers, which, for any given value of σ, will be more easily absorbed in the country with a higher η. Thus, given $\frac{K_t}{L_{s,t}}$ and $\frac{K_t}{L_{u,t}}$, the wage premium will fall by more in the country where skilled labour is more complementary to capital, i.e. France. On the other hand, capital accumulation implies that both $\frac{K_t}{L_{s,t}}$ and $\frac{K_t}{L_{u,t}}$ increase. As capital becomes relatively abundant in terms of both categories of labour, relative factor prices $\frac{r}{w_s}$ and $\frac{r}{w_u}$ both decline, with opposing effects on the wage premium. For a given value of σ, the higher η (as in Denmark) is, the less likely is that the wage premium may increase. Our simulation results suggest that the effect due to increased $\frac{L_{s,t}}{L_{u,t}}$ is more powerful than the effect due to capital accumulation, implying that the overall reduction in the wage premium is higher in the economy where η is smaller, i.e. France.

The unskilled workers' share is another element that enters in expression (10). Capital accumulation encourages the demand for skilled labour relatively more in the country where qualified workers are more complementary to capital, which in turn results in a lower level of unskilled employment and a sharper reduction in their income share in the country where is η smaller (i.e. France).

6.   Policy implications

Over the past three decades labour shares have declined in many European countries. Building on previous literature in this field we have investigated this regularity along two different routes. Firstly, analysing compositional shifts in a descriptive manner, we find that, quite independently of wage moderation, the reduction in labour shares also reflects an increasing weight of those sectors with structurally lower labour shares, together with widespread reductions in the proportion of self-employment in total employment. This suggests that wage-setting policies alone will not be sufficient to reverse the downward trend in labour shares observed in Europe.

The second approach relies on a micro-founded model where the labour share is seen as function of both technological and institutional parameters. The former are captured by factor substitution within the framework of a CES production function defined in terms of capital, skilled and unskilled labour and intermediate inputs, with biased (i.e. capital-augmenting) technical progress also playing a role in the medium term. Institutional determinants include mark-ups, a measure of the bargaining power of un-

Figure 5: *Impact on the labour share of a 5 p.p. reduction in the proportion of unskilled workers in the labour force.*

Simulation results for France and Denmark on the basis of the dynamic model given by expressions (30) to (33).

The upper panel shows the assumed paths for the exogenous variables, namely, the depreciation rate, the population growth rate, the savings rate, the rate of technological progress and the fraction of unskilled labour. The middle and lower panels show the paths for endogenous variables, namely, capital per capita, capital in terms of skilled and unskilled labour, the capital-output ratio, income per capita and the labour share.
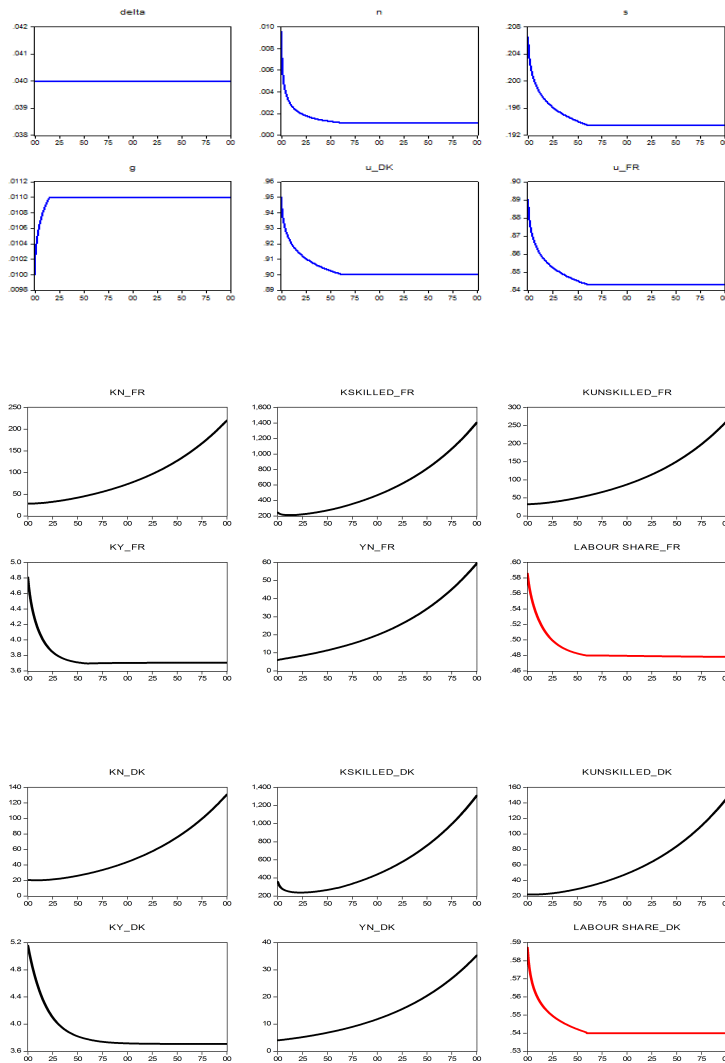
skilled workers and adjustment costs in employment. Our analysis suggests that most of the decline in labour shares is explained by two technological forces, namely capital-augmenting technical progress and the asssumption of capital-skill complementarity, i.e. the fact that capital equipment is complementary to skilled labour but highly substitutive to unskilled labour. Although institutional factors also play a significant role they appear to be of somewhat less importance, explaining a relatively small fraction of the downward movement in labour shares and accounting for only subdued short-term oscillations.

Our simulations point to the forceful implications of the nature of technological progress given unchanged institutional settings. Not only has the labour share fallen over the past three decades, but it may decline further in the future as a result of capital accumulation and an increasing share of skilled labour in total employment. This conclusion could be altered should technical progress be endogenized in the model. However, exploring the complex interactions between the relative supply of skills, institutional settings in labour and product markets and the nature of technological progress is well beyond the scope of this paper.

Appendix 1: Measurement of the labour share

This appendix compares the three measures of the labour share given by expressions (1), (2) and (3) presented in Section 2. We use EU KLEMS data of EU15 Member States covering the period 1970-2004. The sectoral breakdown used in the analysis includes 24 sectors grouped into 9 broadly-defined industries (NACE code in brackets), namely, Agriculture, Hunting, Forestry and Fishing (A-B), Mining and Quarrying (C), Total Manufacturing (D), Electricity, Gas and Water Supply (E), Construction (F), Wholesale and Retail Trade (G), Hotels and Restaurants (H), Transport and Storage and Communication (I), Finance, Insurance, Real Estate and Business Services (J-K). Note that Community Social and Personal Services (L-Q) are excluded, as value added generated by these sectors is merely wage and salary income, so there is no genuine concept of labour share involved. In practical terms, including NACE categories L-Q in the analysis would result in an upward bias of labour's income.

Figure 6 compares the three alternative measures of the labour share (1), (2) and (3). Using (2) instead of (1) results in higher labour shares. This obviously stems from the fact that part of the income earned by the self-employed is remuneration for labour services. This adjustment generally preserves the dynamic patterns in labour shares[35]. The fraction

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[35] Readers should be aware of the fact that Austria has been excluded from the analysis. This is because the imputation of labour income to the self-employed as implied by (2) results in an adjusted labour share exceeding one. This is due to the fact that the correction implied by (2) is not very reliable when the wages for the two types of employment largely differ, which is the case at stake. Specifically, in the case of Austria, equation (2) largely overestimates the income of the self-employed in the 1970s, when these non-employee workers were

of self-employment in total employment (mostly explained by the reduction of the share of self-employed in agriculture) has decreased markedly in Greece, Ireland, France and Spain. This is reflected in the convergence of the two series, (2) and (1), over time. The UK stands out as the only country where the number of employees as a proportion of the total workforce has actually shrunk, implying an increasing gap over time between non-adjusted and adjusted labour shares. In the remaining EU15 countries, the structure of employment in the whole economy has remained broadly stable.

The sectoral correction (i.e. computing labour shares according to (3) instead of (2)) does not change the picture in many EU15 members. Yet, it results in a substantial downward revision of the labour share in several others, namely Greece, Spain, Italy, Portugal, and to a less extent, France and Ireland. Remunerating proprietors' labour at the national average compensation of wage earners tends to largely overestimate the labour share in Greece, Spain and Italy at the beginning of the period, as most of the self-employed in the 1970s were farmers with low earnings.

Appendix 2: Derivation of the labour share under different theoretical assumptions

This appendix shows basic analytical results regarding the specification of the labour share under the various theoretical assumptions adopted in the main text.

Expressions (8) and (9)

Consider a specification for the production function with labour heterogeneity like (5). The marginal productivity of skilled and unskilled labour is respectively given by:

$$Y'_{Ls} = aB_s \left(1 - a\right) \left(B_s L_s\right)^{\rho-1} Y^{\frac{1}{\sigma}} \left[a \left(AK\right)^{\rho} + \left(1 - a\right) \left(B_s L_s\right)^{\rho}\right]^{-\frac{1}{\varepsilon}}$$

$$Y'_{Lu} = \left(1 - \alpha\right) B_u \left(B_u L_u\right)^{-\frac{1}{\sigma}} Y^{\frac{1}{\sigma}}$$

where $\varepsilon = \frac{\sigma\rho}{\sigma(\rho-1)+1}$. Under perfect competition the skill premium (in efficiency units) is:

---

mainly farmers with low earnings. In this country, the share of employees in total employment in the Agriculture sector in 1970 was barely 6%, i.e. atypically low when compared with European standards. This measurement problem tends to be less troublesome when calculating the adjusted labour share on the basis of sectoral data, i.e. following expression (3) in the main text.

Figure 6: *Alternative measures of the labour share, EU15 Member States excl. Austria. Comparison of expressions (1), (2) and (3) in the main text. EU KLEMS data, 1970-2004.*
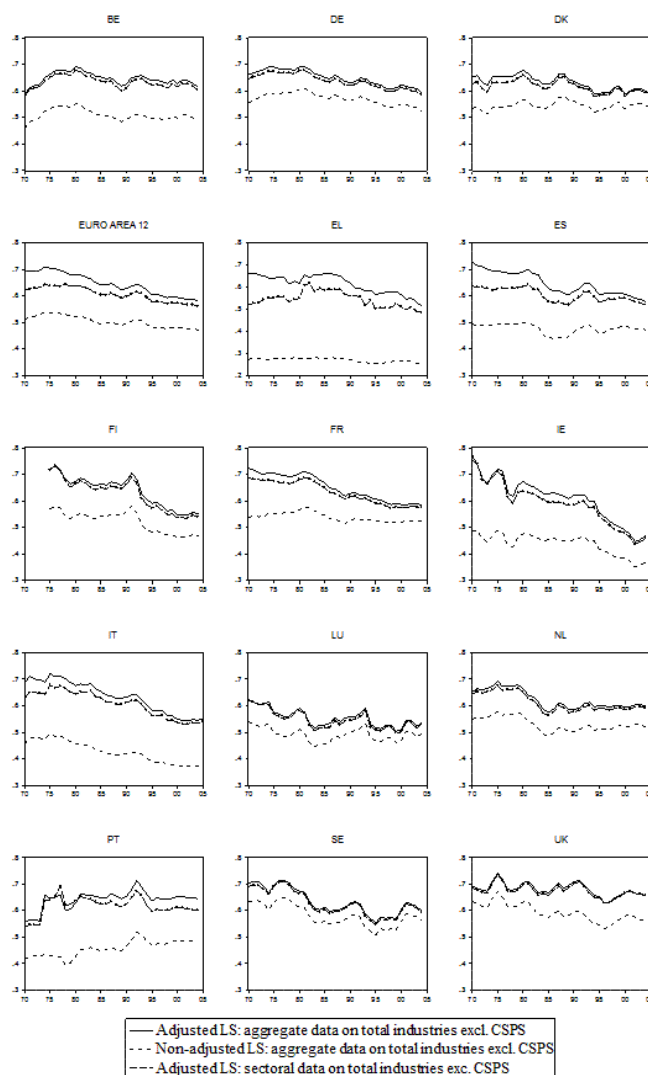
$$\omega = \frac{\frac{w_s^{PC}}{B_s}}{\frac{w_u^{PC}}{B_u}} = \frac{\alpha}{1-\alpha}(1-a)(B_sL_s)^{\rho-1}(B_uL_u)^{\frac{1}{\sigma}}\left[a(AK)^{\rho}+(1-a)(B_sL_s)^{\rho}\right]^{-\frac{1}{\varepsilon}}$$

Under the assumption of perfect competition in the labour market, substitution of the Y_L' for wages in the expression for the labour share gives

$$LS_{PC} = \frac{Y^{\frac{1}{\sigma}}}{Y}\left(\alpha(1-a)(B_sL_s)^{\rho}\left[a(AK)^{\rho}+(1-a)(B_sL_s)^{\rho}\right]^{-\frac{1}{\varepsilon}}+(1-\alpha)(B_uL_u)^{\frac{\sigma-1}{\sigma}}\right)$$

From the skill premium

$$\left[a(AK)^{\rho}+(1-a)(B_sL_s)^{\rho}\right] = \left(\frac{\alpha}{1-\alpha}\right)^{\varepsilon}(1-a)^{\varepsilon}(B_sL_s)^{(\rho-1)\varepsilon}(B_uL_u)^{\frac{\varepsilon}{\sigma}}\omega^{-\varepsilon}$$

Or

$$(B_sL_s)^{\rho} = \frac{1}{1-a}\omega^{-\varepsilon}\left(\frac{\alpha}{1-\alpha}\right)^{\varepsilon}(1-a)^{\varepsilon}(B_sL_s)^{(\rho-1)\varepsilon}(B_uL_u)^{\frac{\varepsilon}{\sigma}}-\frac{a}{1-a}(AK)^{\rho}$$

The capital-output ratio is:

$$\left(A\frac{K}{Y}\right)^{\frac{\sigma-1}{\sigma}} = \frac{(AK)^{\frac{\sigma-1}{\sigma}}}{\alpha\left[a(AK)^{\rho}+(1-a)(B_sL_s)^{\rho}\right]^{\frac{\sigma-1}{\rho\sigma}}+(1-\alpha)(B_uL_u)^{\frac{\sigma-1}{\sigma}}}$$

Solving the above expression for AK one gets:

$$AK = Ak(1-\alpha)^{-\frac{\varepsilon}{\rho}}(B_uL_u)\left\{\alpha^{\varepsilon}(1-a)^{\varepsilon-1}l^{\frac{\sigma-\varepsilon}{\sigma}}\omega^{1-\varepsilon}+(1-\alpha)^{\varepsilon}\right\}^{\frac{\sigma}{(\sigma-1)}}$$

Plugging AK into the expression for $(B_sL_s)^{\rho}$ further above we find:

$$\begin{aligned}(B_sL_s)^{\rho} =\ & \left(\frac{\alpha}{1-\alpha}\right)^{\varepsilon}(1-a)^{\varepsilon-1}(B_sL_s)^{(\rho-1)\varepsilon}(B_uL_u)^{\frac{\varepsilon}{\sigma}}\omega^{-\varepsilon}\\ & -\frac{a}{1-a}\left(\frac{1}{1-\alpha}\right)^{\varepsilon}(Ak)^{\rho}(B_uL_u)^{\rho}\left\{\alpha^{\varepsilon}(1-a)^{\varepsilon-1}l^{\frac{\sigma-\varepsilon}{\varepsilon}}\omega^{1-\varepsilon}+(1-\alpha)^{\varepsilon}\right\}^{\frac{\sigma\rho}{\sigma-1}}\end{aligned}$$

which one may substitute together with the expression above for $\left[a\left(AK\right)^{\rho}+\left(1-a\right)\left(B_sL_s\right)^{\rho}\right]$ into the labour share equation LS_{PC} to obtain, after tedious calculations, expression (8):

$$LS_{PC}=1-\frac{a}{(1-a)}\left(Ak\right)^{\rho}\left\{\alpha^{\varepsilon}\left(1-a\right)^{\varepsilon-1}+\left(1-\alpha\right)^{\varepsilon}l^{\frac{\varepsilon-\sigma}{\sigma}}\omega^{\varepsilon-1}\right\}^{\frac{\sigma\rho}{\varepsilon(\sigma-1)}}$$

Substituting the wage premium by its expression above and developing further, one can get the labour share in terms of the three key ratios, $k=\frac{AK}{Y}$, $l=\frac{B_sL_s}{B_uL_u}$, and $\frac{AK}{B_sL_s}$ as in expression (9):

$$LS_{PC}=1-a\left(Ak\right)^{\rho}\alpha^{\frac{\sigma\rho}{\sigma-1}}\left\{1+\frac{1-\alpha}{\alpha}l^{\frac{1-\sigma}{\sigma}}\left[a\left(\frac{AK}{B_sL_s}\right)^{\rho}+\left(1-a\right)\right]^{\frac{1-\sigma}{\sigma\rho}}\right\}^{\frac{\sigma(\rho-1)+1}{\sigma-1}}$$

Expression (13)

Consider a specification for gross output like (11). The first-order condition for profit maximization with respect to intermediate inputs I is given by:

$$\left(1-\gamma\right)\left(\frac{\widetilde{Y}}{I}\right)^{\frac{1}{\lambda}}=\frac{p_I}{\widetilde{p}}$$

This condition can be solved for I, which yields $I=\left(1-\gamma\right)^{\lambda}\left[\frac{p_I}{\widetilde{p}}\right]^{-\lambda}\widetilde{Y}$. Substituting $\widetilde{Y}$ by (11) in the latter expression and working out I one gets

$$I=\frac{\left[\alpha X^{\frac{\sigma-1}{\sigma}}+\left(1-\alpha\right)\left(B_uL_u\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}}{\left(\frac{1}{\gamma(1-\gamma)^{\lambda-1}}\left[\frac{p_I}{\widetilde{p}}\right]^{\lambda-1}-\frac{1-\gamma}{\gamma}\right)^{\frac{\lambda}{\lambda-1}}}$$

Recall that, according to equation (12), real value added is defined as:

$$Y=\widetilde{Y}-\frac{p_I}{\widetilde{p}}I$$

which can be rewritten as:

$$Y=\left\{\gamma\left(\left[\alpha X^{\frac{\sigma-1}{\sigma}}+\left(1-\alpha\right)\left(B_uL_u\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}\right)^{\frac{\lambda-1}{\lambda}}+\left(1-\gamma\right)I^{\frac{\lambda-1}{\lambda}}\right\}^{\frac{\lambda}{\lambda-1}}-\frac{p_I}{\widetilde{p}}I$$

Substituting in the definition of value added given by the expression above gross output $\widetilde{Y}$ worked out from the first-order condition for intermediate inputs, one can get:

$$Y = \Omega \left[\alpha X^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(B_u L_u\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

where $\Omega = \dfrac{\gamma^{\frac{\lambda}{\lambda-1}}}{\left[1 - \dfrac{(1-\gamma)^\lambda}{\left(\frac{p_I}{p}\right)^{\lambda-1}}\right]^{\frac{1}{\lambda-1}}}$.

Define the labour share in value added under perfect competition as:

$$LS_{PC} = \frac{w_u^{PC} L_u + w_s^{PC} L_s}{Y} = \frac{Y'_{L_u} L_u + Y'_{L_s} L_s}{Y}$$

Following the same steps to find expression (8) one gets:

$$LS_{PC,I} = 1 - \Omega a \left(Ak\right)^\rho \alpha^{\frac{\sigma\rho}{\sigma-1}} \left\{ 1 + \frac{1-\alpha}{\alpha} l^{\frac{1-\sigma}{\sigma}} \left[a \left(\frac{AK}{B_s L_s}\right)^\rho + (1-a) \right]^{\frac{1-\sigma}{\sigma\rho}} \right\}^{\frac{\sigma(\rho-1)+1}{\sigma-1}}$$

where $LS_{PC,I}$ is the labour share with firms operating under perfect competition and facing changes in the relative price of intermediate inputs.

Expression (17)

The marginal productivity of unskilled and skilled labour are respectively given by:

$$Y'_{Ls} = \Omega a B_s \left(1-a\right)\left(B_s L_s\right)^{\rho-1} Y^{\frac{1}{\sigma}} \left[a\left(AK\right)^\rho + (1-a)\left(B_s L_s\right)^\rho \right]^{-\frac{1}{\varepsilon}}$$

$$Y'_{Lu} = \Omega \left(1-\alpha\right) B_u \left(B_u L_u\right)^{-\frac{1}{\sigma}} Y^{\frac{1}{\sigma}}$$

where $\Omega = \dfrac{\gamma^{\frac{\lambda}{\lambda-1}}}{\left[1 - \dfrac{(1-\gamma)^\lambda}{\left(\frac{p_I}{p}\right)^{\lambda-1}}\right]^{\frac{1}{\lambda-1}}}$. In an imperfectly competitive setting, profit maximi-

sation by firms implies:

$$p = \left(1+\mu\right) mc = \left(1+\mu\right) \frac{W_i}{Y'_{L,i}}, i = u, s$$

where W_u, W_s respectively denote nominal wages for unskilled and skilled workers. Rearranging terms:

$$\left(\frac{W_i}{p}\right)^{IC} = w_i^{PC} = \frac{Y'_{L,i}}{(1+\mu)}, i = u, s$$

Define the labour share in value added under imperfect competition as:

$$LS_{IC} = \frac{w_u^{IC} L_u + w_s^{IC} L_s}{Y}$$

where w_u^{IC}, w_s^{IC} respectively denote real wages for skilled and unskilled workers under imperfect competition in the products market. Then substitute real wages for skilled and unskilled workers by the respective marginal productivity of skilled and unskilled labour corrected by the mark-up, thereby getting:

$$LS_{IC,I} = \frac{1}{(1+\mu)} \left(1 - \Omega a \left(Ak\right)^{\rho} \alpha^{\frac{\sigma\rho}{\sigma-1}} \left\{ 1 + \frac{1-\alpha}{\alpha} l^{\frac{1-\sigma}{\sigma}} \left[a \left(\frac{AK}{B_s L_s}\right)^{\rho} + (1-a) \right]^{\frac{1-\sigma}{\sigma\rho}} \right\}^{\frac{\sigma(\rho-1)+1}{\sigma-1}} \right)$$

Appendix 3: Comparative static predictions of the labour share

Expression (9)

Consider the definition of the labour share given by expression (9) in the main text:

$$LS_{PC} = 1 - a \left(Ak\right)^{\rho} \alpha^{\frac{\sigma\rho}{\sigma-1}} \left\{ 1 + \frac{1-\alpha}{\alpha} l^{\frac{1-\sigma}{\sigma}} \left[a \left(\frac{AK}{B_s L_s}\right)^{\rho} + (1-a) \right]^{\frac{1-\sigma}{\sigma\rho}} \right\}^{\frac{\sigma(\rho-1)+1}{\sigma-1}}$$

The derivative of the labour share with respect to A is equal to:

$$
\frac{\partial LS_{PC}}{\partial A} = -a\rho A^{\rho-1} k^{\rho} \alpha^{\frac{\sigma\rho}{\sigma-1}} \left\{ 1 + \frac{1-\alpha}{\alpha} l^{\frac{1-\sigma}{\sigma}} \left[a\left(\frac{AK}{B_s L_s}\right)^{\rho} + (1-a) \right]^{\frac{1-\sigma}{\sigma\rho}} \right\}^{\frac{\sigma(\rho-1)+1}{\sigma-1}} -
$$

$$
- \left(\frac{\sigma(\rho-1)+1}{\sigma-1}\right) a (Ak)^{\rho} \alpha^{\frac{\sigma\rho}{\sigma-1}} \left\{ 1 + \frac{1-\alpha}{\alpha} l^{\frac{1-\sigma}{\sigma}} \left[a\left(\frac{AK}{B_s L_s}\right)^{\rho} + (1-a) \right]^{\frac{1-\sigma}{\sigma\rho}} \right\}^{\frac{\sigma(\rho-1)+1}{\sigma-1}-1}
$$

$$
\left\{ \left(\frac{1-\sigma}{\sigma\rho}\right) \frac{1-\alpha}{\alpha} l^{\frac{1-\sigma}{\sigma}} \left[a\left(\frac{AK}{B_s L_s}\right)^{\rho} + (1-a) \right]^{\frac{1-\sigma}{\sigma\rho}-1} a\rho A^{\rho-1} \left(\frac{K}{B_s L_s}\right)^{\rho} \right\}
$$

A sufficient condition for this derivative to be negative is that $1 < \eta < \sigma$. If $1 < \eta$ ($\rho > 0$), the first term of the derivative is negative. With capital-skill complementarity, i.e. $\eta < \sigma$, the second term of the derivative is also negative.

Expression (10)

Consider the definition of the labour share given by expression (10) in the main text:

$$
LS_{PC} = \theta_{Lu} \left(1 + \frac{w_s^{PC}}{w_u^{PC}} \frac{L_s}{L_u} \right)
$$

Let us reformulate the income share of unskilled workers and the wage ratio in terms of relative factor quantities:

$$
\theta_{Lu} = \frac{1}{1 + \frac{\alpha}{1-\alpha} \left[a\left(\frac{AK}{B_u L_u}\right)^{\rho} + (1-a) l^{\rho} \right]^{\frac{\sigma-1}{\sigma\rho}}}
$$

$$
\frac{w_s^{PC}}{w_u^{PC}} = \frac{B_s}{B_u} \frac{\alpha}{1-\alpha} \left[a\left(\frac{AK}{B_s L_s}\right)^{\rho} + (1-a) \right]^{-\frac{(\rho-1)}{\rho}} \left[a\left(\frac{AK}{B_u L_u}\right)^{\rho} + (1-a) l^{\rho} \right]^{-\frac{1}{\sigma\rho}}
$$

The derivative of the labour share with respect to $\frac{L_s}{L_u}$ is:

$$
\frac{\partial LS_{PC}}{\partial \left(\frac{L_s}{L_u}\right)} = \frac{\partial \theta_{Lu}}{\partial \left(\frac{L_s}{L_u}\right)} \left(1 + \frac{w_s^{PC}}{w_u^{PC}} \frac{L_s}{L_u} \right) + \theta_{Lu} \left[\frac{\partial \left(\frac{w_s^{PC}}{w_u^{PC}}\right)}{\partial \left(\frac{L_s}{L_u}\right)} \frac{L_s}{L_u} + \frac{w_s^{PC}}{w_u^{PC}} \right]
$$

If we now compute the derivates that enter the expression above, we find that:

$$\frac{\partial \theta_{Lu}}{\partial \left(\frac{L_s}{L_u} \right)} = -\frac{\frac{\alpha}{1-\alpha} \frac{\sigma-1}{\sigma\rho} \left[a \left(\frac{AK}{B_u L_u} \right)^\rho + (1-a) l^\rho \right]^{\frac{\sigma-1}{\sigma\rho}-1} (1-a) \rho \frac{B_s}{B_u} l^{\rho-1}}{\left\{ 1 + \frac{\alpha}{1-\alpha} \left[a \left(\frac{AK}{B_u L_u} \right)^\rho + (1-a) l^\rho \right]^{\frac{\sigma-1}{\sigma\rho}} \right\}^2} < 0 \text{ iff } \sigma > 1$$

$$\frac{\partial \left(\frac{w_s^{PC}}{w_u^{PC}} \right)}{\partial \left(\frac{L_s}{L_u} \right)} = -\frac{B_s}{B_u} \frac{\alpha}{1-\alpha} \left[a \left(\frac{AK}{B_s L_s} \right)^\rho + (1-a) \right]^{-\frac{(\rho-1)}{\rho}}$$

$$\left(\frac{1}{\sigma\rho} \right) \left[a \left(\frac{AK}{B_u L_u} \right)^\rho + (1-a) l^\rho \right]^{-\left(\frac{1}{\sigma\rho} \right)-1} (1-a) \rho l^{\rho-1} \frac{B_s}{B_u} < 0, \text{ as } 0 < \sigma < \infty.$$

Plugging the partial derivatives $\frac{\partial \theta_{Lu}}{\partial \left(\frac{L_s}{L_u} \right)}$ and $\frac{\partial \left(\frac{w_s^{PC}}{w_u^{PC}} \right)}{\partial \left(\frac{L_s}{L_u} \right)}$ into $\frac{\partial LS_{PC}}{\partial \left(\frac{L_s}{L_u} \right)}$ we get:

$$\frac{\partial LS_{PC}}{\partial \left(\frac{L_s}{L_u} \right)} = \underbrace{\frac{\partial \theta_{Lu}}{\partial \left(\frac{L_s}{L_u} \right)}}_{(-) \text{ iff } \sigma > 1} \underbrace{\left(1 + \frac{w_s^{PC}}{w_u^{PC}} \frac{L_s}{L_u} \right)}_{(+)} + \underbrace{\theta_{Lu}}_{(+)} \left[\underbrace{\frac{\partial \left(\frac{w_s^{PC}}{w_u^{PC}} \right)}{\partial \left(\frac{L_s}{L_u} \right)}}_{(-)} \underbrace{\frac{L_s}{L_u}}_{(+)} + \underbrace{\frac{w_s^{PC}}{w_u^{PC}}}_{(+)} \right]$$

Thus the sign of the above derivative remain ambiguous. Let us further show that $\frac{\partial LS_{PC}}{\partial \left(\frac{L_s}{L_u} \right)} < 0$ if $\sigma > 1$. To see this, it suffices to develop the second term in the expression for the labour share $LS_{PC} = \theta_{Lu} + \theta_{Lu} \frac{w_s^{PC}}{w_u^{PC}} \frac{L_s}{L_u}$, which yields:

$$LS_{PC} = \theta_{Lu} + l \frac{\frac{\alpha}{1-\alpha} \left[a \left(\frac{AK}{B_s L_s} \right)^\rho + (1-a) \right]^{-\frac{(\rho-1)}{\rho}}}{\frac{1-\alpha}{\alpha} \left[a \left(\frac{AK}{B_u L_u} \right)^\rho + (1-a) l^\rho \right]^{\frac{1}{\sigma\rho}} + \left[a \left(\frac{AK}{B_u L_u} \right)^\rho + (1-a) l^\rho \right]^{\frac{1}{\rho}}}$$

If we now calculate the labour share with respect to $\frac{L_s}{L_u}$, we get:

$$\frac{\partial LS_{PC}}{\partial\left(\frac{L_s}{L_u}\right)} = \frac{\partial\theta_{Lu}}{\partial\left(\frac{L_s}{L_u}\right)} l \frac{\frac{\alpha}{1-\alpha}\left[a\left(\frac{AK}{B_sL_s}\right)^{\rho}+(1-a)\right]^{-\frac{(\rho-1)}{\rho}}}{\frac{1-\alpha}{\alpha}\left[a\left(\frac{AK}{B_uL_u}\right)^{\rho}+(1-a)\,l^{\rho}\right]^{\frac{1}{\sigma\rho}}+\left[a\left(\frac{AK}{B_uL_u}\right)^{\rho}+(1-a)\,l^{\rho}\right]^{\frac{1}{\rho}}}$$

$$-\theta_{Lu}\frac{l\frac{\alpha}{1-\alpha}\left[a\left(\frac{AK}{B_sL_s}\right)^{\rho}+(1-a)\right]^{-\frac{(\rho-1)}{\rho}}\left\{\begin{array}{c}\frac{1-\alpha}{\alpha}\frac{1}{\sigma}\left[a\left(\frac{AK}{B_uL_u}\right)^{\rho}+(1-a)\,l^{\rho}\right]^{\frac{1}{\sigma\rho}-1}(1-a)\,\rho\frac{B_s}{B_u}l^{\rho-1}+\\\left[a\left(\frac{AK}{B_uL_u}\right)^{\rho}+(1-a)\,l^{\rho}\right]^{\frac{1}{\rho}-1}(1-a)\frac{B_s}{B_u}l^{\rho-1}\end{array}\right\}}{\left\{\frac{1-\alpha}{\alpha}\left[a\left(\frac{AK}{B_uL_u}\right)^{\rho}+(1-a)\,l^{\rho}\right]^{\frac{1}{\sigma\rho}}+\left[a\left(\frac{AK}{B_uL_u}\right)^{\rho}+(1-a)\,l^{\rho}\right]^{\frac{1}{\rho}}\right\}^{2}}$$

The sign of the first term in the right-hand side is positive iff $\sigma < 1$ and negative iff $\sigma > 1$. The second term in the right-hand side is always positive. Thus, the derivative of the labour share with respect to the relative supply of skilled workers is negative if $\sigma > 1$.

The derivative of the labour share with respect to $\frac{AK}{B_uL_u}$ is:

$$\frac{\partial LS_{PC}}{\partial\left(\frac{AK}{B_uL_u}\right)} = \frac{\partial\theta_{Lu}}{\partial\left(\frac{AK}{B_uL_u}\right)}\left(1+\frac{w_s^{PC}}{w_u^{PC}}\frac{L_s}{L_u}\right)+\theta_{Lu}\left[\frac{\partial\left(\frac{w_s^{PC}}{w_u^{PC}}\right)}{\partial\left(\frac{AK}{B_uL_u}\right)}\frac{L_s}{L_u}\right]$$

If we now compute the derivates that enter the expression above, we find that:

$$\frac{\partial\theta_{Lu}}{\partial\left(\frac{AK}{B_uL_u}\right)} = -\frac{\frac{\alpha}{1-\alpha}\frac{\sigma-1}{\sigma\rho}\left[a\left(\frac{AK}{B_uL_u}\right)^{\rho}+(1-a)\,l^{\rho}\right]^{\frac{\sigma-1}{\sigma\rho}-1}a\rho\left(\frac{AK}{B_uL_u}\right)^{\rho-1}}{\left\{1+\frac{\alpha}{1-\alpha}\left[a\left(\frac{AK}{B_uL_u}\right)^{\rho}+(1-a)\,l^{\rho}\right]^{\frac{\sigma-1}{\sigma\rho}}\right\}^{2}} < 0 \text{ iff } \sigma > 1$$

$$\frac{\partial\left(\frac{w_s^{PC}}{w_u^{PC}}\right)}{\partial\left(\frac{AK}{B_uL_u}\right)} = -\frac{B_s}{B_u}\frac{\alpha}{1-\alpha}\left[a\left(\frac{AK}{B_sL_s}\right)^{\rho}+(1-a)\right]^{-\frac{(\rho-1)}{\rho}}$$

$$\frac{1}{\sigma\rho}\left[a\left(\frac{AK}{B_uL_u}\right)^{\rho}+(1-a)\,l^{\rho}\right]^{-\frac{1}{\sigma\rho}-1}a\rho\left(\frac{AK}{B_uL_u}\right)^{\rho-1} < 0, \text{ as } 0 < \sigma < \infty$$

Plugging the partial derivatives $\dfrac{\partial \theta_{Lu}}{\partial \left(\frac{AK}{B_u L_u}\right)}$ and $\dfrac{\partial \left(\frac{w_s^{PC}}{w_u^{PC}}\right)}{\partial \left(\frac{AK}{B_u L_u}\right)}$ into $\dfrac{\partial LS_{PC}}{\partial \left(\frac{AK}{B_u L_u}\right)}$ we get:

$$\frac{\partial LS_{PC}}{\partial \left(\frac{AK}{B_u L_u}\right)} = \underbrace{\frac{\partial \theta_{Lu}}{\partial \left(\frac{AK}{B_u L_u}\right)}}_{(-)\,\text{iff }\sigma>1} \underbrace{\left(1 + \frac{w_s^{PC}}{w_u^{PC}}\frac{L_s}{L_u}\right)}_{(+)} + \underbrace{\theta_{Lu}}_{(+)} \left[\underbrace{\frac{\partial \left(\frac{w_s^{PC}}{w_u^{PC}}\right)}{\partial \left(\frac{AK}{B_u L_u}\right)}}_{(-)} \underbrace{\frac{L_s}{L_u}}_{(+)}\right]$$

Thus $\dfrac{\partial LS_{PC}}{\partial \left(\frac{AK}{B_u L_u}\right)} < 0$ iff $\sigma > 1$.

The derivative of the labour share with respect to $\frac{AK}{B_s L_s}$ is:

$$\frac{\partial LS_{PC}}{\partial \left(\frac{AK}{B_s L_s}\right)} = \frac{\partial \theta_{Lu}}{\partial \left(\frac{AK}{B_s L_s}\right)} \left(1 + \frac{w_s^{PC}}{w_u^{PC}}\frac{L_s}{L_u}\right) + \theta_{Lu}\left[\frac{\partial \left(\frac{w_s^{PC}}{w_u^{PC}}\right)}{\partial \left(\frac{AK}{B_s L_s}\right)} \frac{L_s}{L_u}\right]$$

If we now compute the derivatives that enter the expression above, we find that:

$$\frac{\partial \theta_{Lu}}{\partial \left(\frac{AK}{B_s L_s}\right)} = 0$$

$$\frac{\partial \left(\frac{w_s^{PC}}{w_u^{PC}}\right)}{\partial \left(\frac{AK}{B_s L_s}\right)} = -\frac{B_s}{B_u}\frac{\alpha}{1-\alpha}\left[a\left(\frac{AK}{B_u L_u}\right)^\rho + (1-a)\,l^\rho\right]^{-\frac{1}{\sigma \rho}}$$

$$\frac{\rho-1}{\rho}\left[a\left(\frac{AK}{B_s L_s}\right)^\rho + (1-a)\right]^{-\left(\frac{\rho-1}{\rho}\right)-1} a\rho\left(\frac{AK}{B_s L_s}\right)^{\rho-1} \quad > \quad 0\,(<0)\,if\,\rho<1\,(\rho>1).$$

Thus $\dfrac{\partial LS_{PC}}{\partial \left(\frac{AK}{B_s L_s}\right)} > 0$ iff $\eta > 0$ which is always the case.

Expression (13)

Consider the definition of the labour share given by expression (9) in the main text:

$$LS_{PC,I} = 1 - \Omega a\,(Ak)^\rho\,\alpha^{\frac{\sigma \rho}{\sigma-1}}\left\{1 + \frac{1-\alpha}{\alpha}l^{\frac{1-\sigma}{\sigma}}\left[a\left(\frac{AK}{B_s L_s}\right)^\rho + (1-a)\right]^{\frac{1-\sigma}{\sigma \rho}}\right\}^{\frac{\sigma(\rho-1)+1}{\sigma-1}}$$

where $\Omega = \dfrac{\gamma^{\frac{\lambda}{\lambda-1}}}{\left[1 - \dfrac{(1-\gamma)^\lambda}{\left(\frac{p_I}{\tilde{p}}\right)^{\lambda-1}}\right]^{\frac{1}{\lambda-1}}}.$

$$\frac{\partial LS_{PC}}{\partial \left(\frac{p_I}{\tilde{p}}\right)} = -\frac{\Omega}{\partial \left(\frac{p_I}{\tilde{p}}\right)} a \left(Ak\right)^\rho \alpha^{\frac{\sigma\rho}{\sigma-1}} \left\{1 + \frac{1-\alpha}{\alpha} l^{\frac{1-\sigma}{\sigma}} \left[a\left(\frac{AK}{B_s L_s}\right)^\rho + (1-a)\right]^{\frac{1-\sigma}{\sigma\rho}}\right\}^{\frac{\sigma(\rho-1)+1}{\sigma-1}}$$

$$\frac{\partial LS_{PC,I}}{\partial \left(\frac{p_I}{\tilde{p}}\right)} = \gamma^{\frac{\lambda}{\lambda-1}} (1-\gamma)^\lambda \frac{1}{\left(\frac{p_I}{\tilde{p}}\right)^\lambda} \left[1 - \frac{(1-\gamma)^\lambda}{\left(\frac{p_I}{\tilde{p}}\right)^{\lambda-1}}\right]^{\frac{1}{\lambda-1}} a \left(Ak\right)^\rho \alpha^{\frac{\sigma\rho}{\sigma-1}}$$

$$\left\{1 + \frac{1-\alpha}{\alpha} l^{\frac{1-\sigma}{\sigma}} \left[a\left(\frac{AK}{B_s L_s}\right)^\rho + (1-a)\right]^{\frac{1-\sigma}{\sigma\rho}}\right\}^{\frac{\sigma(\rho-1)+1}{\sigma-1}}$$

It then follows that $\frac{\partial LS_{PC,I}}{\partial \left(\frac{p_I}{\tilde{p}}\right)} > 0$ iff $\left[1 - \frac{(1-\gamma)^\lambda}{\left(\frac{p_I}{\tilde{p}}\right)^{\lambda-1}}\right] > 0$, which is always the case. To see this, recall that:

$$\tilde{Y} = Y + \frac{p_I}{\tilde{p}} I$$

which implies that

$$I = \frac{Y \left(\frac{p_I}{\tilde{p}}\right)^{-\lambda} (1-\gamma)^\lambda}{\left[1 - \frac{(1-\gamma)^\lambda}{\left(\frac{p_I}{\tilde{p}}\right)^{\lambda-1}}\right]}$$

thus, obviously $\left[1 - \frac{(1-\gamma)^\lambda}{\left(\frac{p_I}{\tilde{p}}\right)^{\lambda-1}}\right] > 0$ as $I > 0$.

References

Acemoglu, D., Aghion, P. & Violante, G. L, (2001), "Deunionization, Technical Change and Inequality", CEPR Discussion Papers 2764.

Bentolila, S. & Bertola, G., (1990), "Firing Costs and Labour Demand: How Bad is Eurosclerosis?", Review of Economic Studies, 57, 381-402.

Bentolila. S. & Saint-Paul, G., (2003), "Explaining Movements in the Labor Share", Contributions to Macroeconomics, Berkeley Electronic Press, vol. 3(1), pages 1103-1103.

Blanchard, O. & Wolfers, J., (2000), "The role of shocks and institutions in the rise of European unemployment: the aggregate evidence", Economic Journal, Royal Economic Society, vol. 110(462), pages C1-33, March.

Blanchard, O. & Giavazzi, F., (2003), "Macroeconomic Effects of Regulation and Deregulation in Goods and Labor Markets", Quarterly Journal of Economics 118, 3, 879-907.

Boeri, T. & Cramer, U., (1992), "Employment Growth, Incumbents and Entrants: Evidence from Germany", International Journal of Industrial Organisation, No.10.

Booth, A.L., (1995), The Economics of the Trade Union, (Cambridge).

Bruno, M. & Sachs, J., (1985), Economics of Worldwide Stagflation, Cambridge, Mass., Harvard University Press.

Caselli, F. & Coleman, W.J, (2001), "Cross-Country Technology Diffusion: The Case of Computers", CEPR Discussion Papers 2744, C.E.P.R.

Fallon, P. R. & Layard, R., (1975) "Capital-skill complementarity, income distribution and output accounting", Journal of political economy, 83 (2), pages 279-302. ISSN 0022-3808.

De Serres, A., Scarpetta, S., & de la Maisonneuve, C., (2002), "Sectoral Shifts in Europe and the United States: How they affect Aggregate Labour Shares and the Properties of Wage Equations", OCDE. Economic Department Working Paper 326.

European Commission (2007), "Labour market and wage developments in 2006", European Economy, No. 4, 2007.

European Commission (2009), "The 2009 Ageing Report: Economic and Budgetary Projections for the EU-27 Member States", European Economy, No. 2, 2009.

Garibaldi, P., Konings, J. & Pissarides, C.A., (1997), "Gross Job Reallocation and Labour Market Policy", Unemployment Policy: Government Options for the Labour Market. Edited by D.J. Snower and G. de la Dehesa, Cambridge University Press, Cambridge.

Goldin, C. & Katz, L. F., (1996), "The Origins of Technology-Skill Complementarity", NBER Working Papers 5657, National Bureau of Economic Research, Inc.

Gollin, D., (2002), "Getting Income Shares right", Journal of Political Economy, 110(2), 458–74.

Gordon, R., (2005) "Where Did the Productivity Growth Go?", NBER Working Paper 11842.

Griliches, Z., (1969), "Capital-Skill Complementarity", The Review of Economics and Statistics, MIT Press, vol. 51(4), pages 465-68, November.

Hamermesh, D.S., (1993a), Labour demand, Princeton University Press.

Hamermesh, D.S. & Pfann, G.A., (1996), "Adjustment Costs in Factor Demand", Journal of Economic Literature, Vol. XXXIV, September 1996, pp. 1264-1292.

Hansen, G.D. & Prescott, E.E., (2005): "Capacity Constraints, Aymmetries, and the Business Cycle", Review of Economic Dynamics, 2005(1), 850-865.

Harrison, A., (2003), "Has Globalization Eroded Labor's Share? Evidence from Cross-Country Data", Working Paper, University of California-Berkeley.

Hicks, J.R., (1963), The Theory of Wages, 2nd ed., Macmillan, London.

Jaramillo, F., Schiantarelli, F. & Sembenelli, A., (1993), "Are Adjustment Costs for Labour Asymmetric? An Econometric Test on Panel Data for Italy", Review of Economics and Statistics, November 1993, 74(4), pp. 640-48.

Jones, C. I., (2003), "Growth, Capital Shares, and a New Perspective on Production Functions", Working Paper.

Kessing, S., (2001), "A note on the determinants of labour share movements", 2003, Economics Letters, 81 (1), 9-12.

Klump R., McAdam P. & Willman A., (2007), "Factor Substitution and Factor Augmenting Technical Progress in the US", Review of Economics and Statistics (2007) 89(1):183–92.

Klump, R., & Preissler, H. (2000), 'CES Production Functions and Economic Growth', Scandinavian Journal of Economics, 102(1), 41–56.

Krusell, P., Ohanian, L. E., Rios-Rull, J.V. & Violante, G.L., (2000), "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis", Econometrica, Econometric Society, vol. 68(5), pages 1029-1054, September.

Lagarde, S., E. Maurin & C. Torelli (1994), "Créations et Suppressions d'emplois en France: une étude de la période 1984-1992", Economie et Prévision, 67-88.

de La Grandville, O., (1989), "In quest of the Slutsky diamond", American Economic Review, 79(3), 468-481.

Leontief, W., (1947), "Wages, Profits, Prices and Taxes", Dun's Review.

McAdam, P. & Willman, A., (2004b), "Supply, Factor Shares and Inflation Persistence: Re-examining Euro-area New-Keynesian Phillips Curves", Oxford Bulletin of Economics and Statistics, 66(9), 637–70.

Messina, J., & Vallanti, G., (2006), "Job Flow Dynamics and Firing Restrictions: Evidence from Europe", IZA DP No. 2045.

Oliveira, J., Scarpetta, S. & Pilat, D., (1996), "Mark-Up Ratios in Manufacturing Industries. Estimates for 14 OECD countries", OECD Economics Department Working Papers, No. 162.

OECD, (1999), Employment Outlook (ch. 2), Paris.

OECD, (2007), Employment Outlook (ch. 3), Paris.

Papageorgiou, C. & Saam, M., "Two-Level CES Production Technology in the Solow and Diamond Growth Models", Departmental Working Papers 2005-07, Department of Economics, Louisiana State University.

Palm, F.C., & Pfann, G.A. (1993), "Asymmetric adjustment costs in non-labour demand models for the Netherlands and U.K. manufacturing sectors", Review of economic studies, 60, 297-312.

Piketty, T., (2007), "Income, wage and wealth inequality in France, 1901-98", in A B Atkinson and T Piketty editors, *Top incomes over the Twentieth Century*, Oxford University Press, Oxford.

Piketty, T. & Saez, E., (2007), "Income, wage and wealth inequality in the United States, 1913-02", in A B Atkinson and T Piketty editors, *Top incomes over the Twentieth Century*, Oxford University Press, Oxford.

Ríos-Rull, V., & Santaeulàlia-Llopis, R., (2007), "Redistributive Shocks and Allocative Shocks", Mimeo, University of Pennsylvania.

Sato, K., (1967), "A two-level constant elasticity of substitution production function", Review of Economic Studies 34 (1967, April).

Saito, M., (2004), "Armington elasticities in intermediate inputs trade: a problem in using multilateral trade data", IMF Working Paper, 2004, No. 22.

Trigari, A., (2004), "Equilibrium unemployment, job flows and inflation dynamics", Working Paper Series 304, European Central Bank.

Ventura, J., (1997), "Growth and Interdependence", Quarterly Journal of Economics 62, 57-84.

Weder, M. & Grubel, H.G., (1993), The New Growth Theory and Cosean Economics: Institutions to Capture Externalities, Weltwirtschaftliches Archiv 129, 488-513.

Young, A.T., (2004): "Labour's Share Fluctuations, Biased Technological Change, and the Business Cycle", Review of Economic Dynamics, 2004(7), 916-931.

Zuleta, H., & Young, A. T.,(2007), "Labor's Shares – Aggregate and Industry: Accounting for both in a model of unbalanced growth with induced innovation", Documentos de trabajo 003105, Universidad del Rosario, Facultad de Economia.